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Laser Beams with Axially Symmetric Polarization

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ABSTRACT. The analysis of the vector wave equation was conducted. The class of self-similar solutions with inhomogeneous polarization corresponding to the resonator modes is deduced. The modes with inhomogeneous polarization can be selected by a diffraction element with polarization selectivity used as one of the resonator mirrors. Diffraction elements with high polarization selectivity about 100% are necessary for generating "pure" radially polarized modes. The radially polarized beam provides higher energy efficiency (the product of the depth of the cut by cutting velocity) for laser cutting metals than the circular polarized main mode does under the same conditions. The two limiting cases of resonance absorption on the spherical plasma target could be realized using axially polarized beams: the resonance absorption is maximum in the case of radial polarization and doesn't occur in the case of azimuthal polarization.

1. INTRODUCTION

Polarization is one of the most important characteristics of laser radiation. While determining a polarization state of the beam one can speak about type of polarization at the point of the beam cross section, homogeneity of ellipsometrical parameters over the beam cross section and stability of polarization characteristics in time.

The radiation of the modern gas lasers has homogeneous polarization, i.e. ellipsometrical parameters over the cross-section of the laser beam are constant. As a rule, the element determining the stable direction of plane polarization is placed inside the laser resonator. This can be a Brewster window in the low-power lasers or one or several turning mirrors in high-power lasers.

From the nowadays viewpoint, the conventional types of polarization have substantial disadvantages. In the case of linear polarization, the parameters of the beam interaction with the matter depend upon the direction of polarization. In the case of circular polarization, these parameters are time averaged, i.e., not optimum from viewpoint either of minimum losses or maximum absorption.

The modes with inhomogeneous polarization, radial or azimuthal, are known in the laser resonator theory. In the case of radial (azimuthal) polarization the direction of the electrical vector in the plane of the beam cross section is parallel (perpendicular) to the radial direction. The axial symmetrically polarized mode TEM_{01*} results from superposition of the two linearly polarized modes TEM_{01} turned around the beam axis at 90°, if their planes of polarization are perpendicular to each other and phase shift equals zero [1].

This designation is often used for the mode with conventional, homogeneous polarization, the so called doughnut mode. One also suggests that the ring intensity distribution of the doughnut mode results from superposition of the two TEM_{01} modes. Nevertheless it is easy to show that the intensity distribution of the resulted mode can not possess a ring shape at any relative phase shift in the two coherent modes TEM_{01}. The ring intensity distribution of the doughnut mode can be explained as time averaging of random orientation of the one TEM_{01} mode. So the designation TEM_{01*} is more correct for modes with radial or azimuthal polarization.

In the present study the modes with inhomogeneous polarization are considered. The problem of selecting such modes is analyzed. The effective methods of selecting these modes using special diffraction elements inside a laser resonator, perspectives of application of radially and azimuthally polarized beams are discussed.

2. ANALYSIS OF THE VECTOR WAVE EQUATION

In the general case a free space laser beam is described by the vector wave equation:

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial \mathbf{E}}{\partial r}+\frac{1}{r^2}\frac{\partial^2 \mathbf{E}}{\partial \varphi^2}+\frac{\partial^2 \mathbf{E}}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial c^2}=0\,. \tag{1}$$

Using famous presentation for beams directed along z axis:

$$\mathbf{E}(r,\,\varphi,\,z) = \mathbf{E}(r,\,\varphi,\,z)\cdot\exp[i(kz\text{-}\omega t)], \tag{2}$$

for paraxial beams the equation (1) is transformed to:

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial \mathbf{E}}{\partial r}+\frac{1}{r^2}\frac{\partial^2 \mathbf{E}}{\partial \varphi^2}+2ik\frac{\partial \mathbf{E}}{\partial z}=0\,. \tag{3}$$

In the case of linear polarization with homogeneous distribution of ellipsometrical parameters over the beam cross section, the expression (3) becomes a scalar equation. The self-similar solutions of this equation are the well known Laguerre Gauss beams [2].

3. EQUATION FOR BEAMS WITH AXIALLY SYMMETRIC POLARIZATION

The above mentioned TEM_{01*} modes with radial and azimuthal types of polarization belong to the class of modes with axially symmetrical polarization (ASP). The electrical vector of an ASP mode intersects the radial direction at an arbitrary angle which is constant over the beam cross section. In the situations when the waist radius is much bigger than the wavelength we can neglect the longitudinal component of field. Then the solution of the equation (3) has the form:

$$\mathbf{E}(r,z)=\mathbf{n}(\varphi)\cdot E(r,z). \tag{4}$$

Here $\mathbf{n}(\varphi)$ is the unit vector in the plane perpendicular to z-axis, for which the following expressions are valid:

$$\frac{\partial \mathbf{n}}{\partial \varphi}=-\mathbf{n}^*; \qquad \frac{\partial^2 \mathbf{n}}{\partial \varphi^2}=-\mathbf{n}; \qquad \frac{\partial \mathbf{n}}{\partial r}=0; \qquad \frac{\partial \mathbf{n}}{\partial z}=0 \tag{5}$$

n*(φ) is the perpendicular to **n**(φ) unit vector.

Taking into account (4),(5), the equation (3) can be transformed:

$$E''_{rr} + \frac{1}{r}E'_{r} - \frac{1}{r^2}E + 2ikE'_{z} = 0 \qquad (6)$$

Solving the equation (6) by the method of separation of variables, one can obtain the solution in the general form:

$$E(r,z) = A(\beta^2)\cdot J_1(\beta r)\cdot \exp\left(\frac{i}{2k}\beta^2 z\right) \qquad (7)$$

where $A(\beta^2)$ is an arbitrary function of the separation parameter β^2, J_1 is the first order Bessel function.

4. ASP LAGUERRE GAUSS BEAMS AS SUPERPOSITION OF BESSEL BEAMS

Choose the form of the function $A(\beta^2)$ as the following:

$$A(\beta^2)=\exp(-c^2\beta^2)\cdot L^1_p(\beta^2)\cdot(-1)^p. \qquad (8)$$

Here c^2=const, $L^1_p(\beta^2)$ are Laguerre polynomials with the azimuthal index 1 and radial index p.

Integrating the expression (7) over β^2 between 0 and ∞ and applying integral formulae [2,4], one can show that the intensity of the beam with axially symmetric polarization has the form:

$$|u(r,z)|^2 = \mathrm{const}\cdot\frac{r^2}{w^2(z)}\cdot\left(L^1_p\left(\frac{r^2}{w^2(z)}\right)\right)^2\cdot\exp\left(\frac{-2r^2}{w^2(z)}\right), \qquad (9)$$

which is analogous with the expression for intensity distribution of Laguerre Gauss modes TEM_{P1*}. The function $A(\beta^2)$ determines the weights of Bessel beams with axially symmetric polarization superposition of which builds the Laguerre Gauss beam with the same type of polarization. For the Laguerre Gauss beams are a system of orthogonal functions, there is a representation of an Bessel beam through these functions. As shown experimentally in the study [5], the distance, at which diffraction free property remains, depends upon an approximation degree of a beam distribution to the ideal Bessel distribution.

5. ASP BEAMS IN THE GENERAL CASE

The analysis of ASP beams based on the equation (6) was made on the assumption that the intensity distribution doesn't depend upon φ (4). Besides the self-similar solutions were considered. In the general case, we are returning to the equation (3) to show that beams with axially symmetric polarization and intensity distribution dependent on φ can not exist.

After separating the variables in (3), the equation for the electrical field component dependent on φ has the form:

$$\frac{\partial^2}{\partial \varphi^2}\Phi\,\mathbf{n} = -m^2\Phi\,\mathbf{n}. \tag{10}$$

This expression transforms to the following equation:

$$\Phi''\mathbf{n} - 2\Phi'\mathbf{n}^* + \Phi(m^2-1)\mathbf{n} = 0, \tag{11}$$

where m is parameter of separation; **n**, **n*** are unit vectors defined in (5).

One can conclude from (11) that Φ is constant and m=1. Thus, any beam with axially symmetric polarization must have an axially symmetric intensity distribution. Propagating in free space such beams with arbitrary radial intensity distribution retain their state of polarization.

6. THE VECTOR SUPERPOSITION OF THE TWO TEM_{01} MODES

The modes with radial and azimuthal types of polarization, R-TEM_{01*} and A-TEM_{01*} correspondingly, are one of the results of vector superposition of two linearly polarized modes TEM_{01}. As shown below, the general case of such a superposition is important from the viewpoint of selecting R-TEM_{01*} and A-TEM_{01*} modes.

In general case:

$$\mathbf{E}(r,\varphi,z) = a_1\,\mathbf{n}_1\,E_1(r,\varphi-\beta,z) + a_2\,\mathbf{n}_2\,E_2(r,\varphi+\beta,z), \tag{12}$$

where a_1, a_2 are arbitrary complex constants; 2β is the angle of relative orientation of the modes; $\mathbf{n}_1$, $\mathbf{n}_2$ are unit vectors indicating the direction of linear polarization.

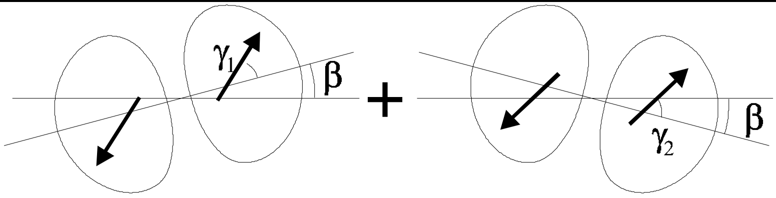
The vector superposition (12) at arbitrary β, a_i, $\mathbf{n}_i$ is a mode too. The most interesting cases are shown in Table 1 provided that $|a_1| = |a_2|$.

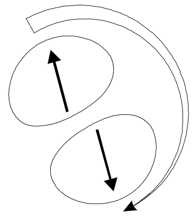
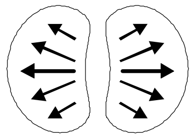
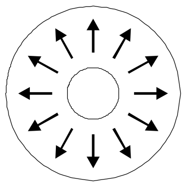
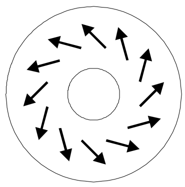
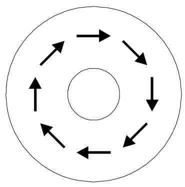
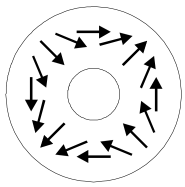
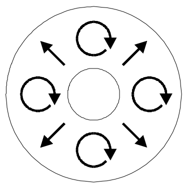
7. SELECTING MODES WITH INHOMOGENEOUS POLARIZATION

The linearly polarized spiral mode is of interest deviating from the theme of the present study. Taking into account the practical importance of the R-TEM_{01*} and A-TEM_{01*} modes (Table 1, rows 3,5) we consider conditions of selecting these modes.

Note that the tasks of selecting ASP modes and fixing the direction of polarization differ substantially. As a rule, one solves the second task by means of installing an optical element with polarization selectivity less than 1% inside the resonator. Requirements to devices for selecting ASP modes are higher.

The method for selecting such modes using diffraction optical element (DOE) with axially symmetric groove structure as a rear mirror of the resonator is offered in the study [6]. A DOE with local polarization selectivity 20% was used in CO_2-laser with output power 2 kW.

<table>
<tr><td colspan="8">

φ - azimuthal angle,
Δ_{12} - two modes relative phase shift.</td></tr>
<tr><th>N</th><th>2β</th><th>Δ_{12}</th><th>γ_1</th><th>γ_2</th><th>Polarization scheme</th><th>Field amplitude</th><th>Comments</th></tr>
<tr><td>1</td><td>π/2</td><td>π/2</td><td>γ_1</td><td>γ_1-π/2</td><td></td><td>$E(r, \varphi)\, e^{i(\omega t-\varphi)}$</td><td>Spiral mode with plane polarization</td></tr>
<tr><td>2</td><td>π/3</td><td>0</td><td>0</td><td>0</td><td></td><td>$E(r, \varphi)\, e^{i\omega t}$</td><td>Intermediate state between modes L-TEM_{01} и R-TEM_{01*}</td></tr>
<tr><td>3</td><td rowspan="4">π/2</td><td rowspan="4">0</td><td>0</td><td>0</td><td></td><td>$E(r)\, e^{i\omega t}$</td><td>Radially polarized mode R-TEM_{01*}</td></tr>
<tr><td>4</td><td>π/4</td><td>π/4</td><td></td><td>$E(r)\, e^{i\omega t}$</td><td>Mode with the angle between vector of electric field and radius π/4</td></tr>
<tr><td>5</td><td>π/2</td><td>π/2</td><td></td><td>$E(r)\, e^{i\omega t}$</td><td>Azimuthally polarized mode A-TEM01*</td></tr>
<tr><td>6</td><td>0</td><td>π</td><td></td><td>$E(r)\, e^{i\omega t}$</td><td>Mode with direction of electric field equal to π/2-φ</td></tr>
<tr><td>7</td><td>π/2</td><td>π/2</td><td>0</td><td>0</td><td></td><td colspan="2">At φ=k·π/2,
circular polarization.
At φ= k·π/2+π/4
plane polarization.
k=0, 1, 2, 3</td></tr>
</table>

The losses of "classical" linearly polarized modes on the DOE with axially symmetric groove structure were also calculated. The TEM_{P1*} modes possess minimum losses on such DOE provided that plane of polarization is directed along the axis that crosses the points of maximum intensity over the cross section (γ_1=0 γ_2=0, Table 1). For the modes with an arbitrary angle of relative orientation can exist (Table 1, row 2), generation of the ASP modes possessing ideal ring intensity distribution depends upon polarization selectivity of the DOE.

8. INTERACTION OF ASP BEAMS WITH METALS

It is well known that parameters of laser processing metals depend upon polarization. Ultimate cutting parameters for the linearly polarized beam are worse than for circularly polarized beam although absorption on the front of the cut is maximum in the first case and average between maximum and the minimum values in the second case. This fact is explained in the study [7], where it is shown that absorption of radiation on the cut surface, which is optimum for technological goals, must be axially symmetric and maximum. The radially polarized beams meet these requirements. A possibility of doubling laser cutting efficiency on account of radial polarization used instead of circular polarization was also predicted.

Today one obtain the best results in laser cutting metals using the main mode with circular polarization. We are applying to Fig. 1 to compare efficiency of laser cutting with this mode and the R-TEM_{01*} mode. As is well known, the intensities of radiation of these modes differ at a maximum by "e" times. The radius of the beam area which embraces 86% of the power is larger by 1.32 times for the R-TEM_{01*} mode than for the main mode (curves "a", Fig. 1). Therefore the technological application of the circularly polarized TEM_{01*} mode is limited in comparison with the main mode having the same type of polarization.

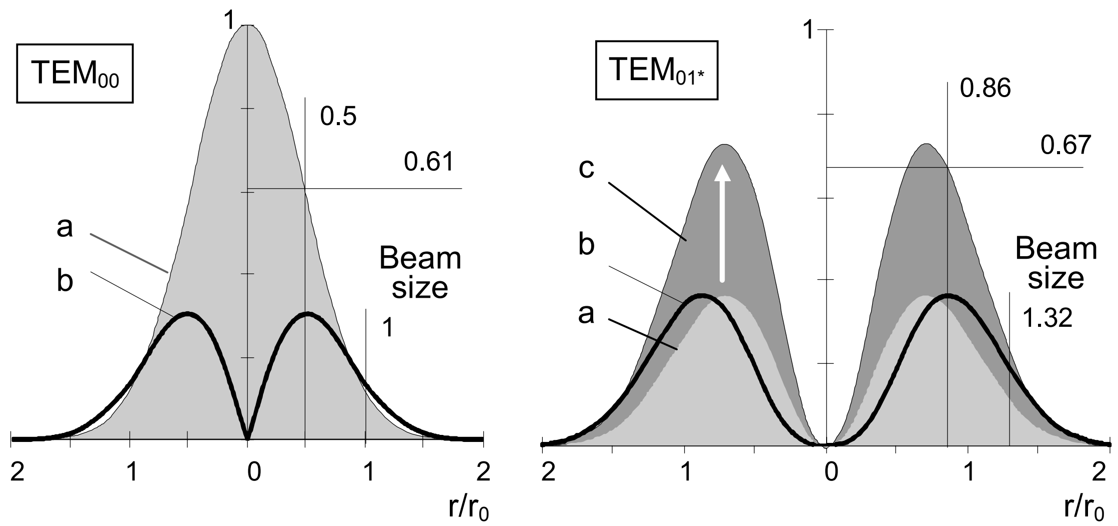


Fig.1. The radial dependence of radiation intensity I(r)/I0 (a); integrand r·I(r) (b); doubled "effective" intensity in the case of radial polarization (c).

Nevertheless the estimations based on the comparison of intensities I(r) at a maximum are inexact.

The laser beam power P may be written as $P = 2\pi \int_0^\infty r \cdot I(r) \cdot dr$. The integrand I(r)r as a function of r is shown in Fig. 1 (curves "b"). The central zone of the TEM_{00} mode possessing an intensity maximum doesn't contribute "energy input" to the beam power. The value $I/I_0=0.61$ corresponding to the maximum of the integrand I(r)r should be considered as the characteristic beam intensity. Maximum values of I(r) and I(r)r for the TEM_{01*} mode differ insignificantly.

The doubling of absorption coefficient in the case of radial polarization at large angles of incidence can be taken into account as doubling of the beam intensity (curve "c", Fig. 1) at which the size of the beam remains the same.

The estimations presented in Table 2 show that the R-TEM_{01*} mode cutting possesses a wider kerf and higher efficiency (the product of depth of the cut by cutting velocity) even in comparison with the circularly polarized TEM_{00} mode. It is difficult to dispute the advantages of the main mode in precise laser cutting used, for example, for making souvenirs at which the sharp focusing is important. The use of the R-TEM_{01*} mode is preferable for the most technological laser complexes equipped with high power lasers and aimed at effective cutting metals.

Mode	TEM_{00}	TEM_{01*}
Polarization	Circular	Radial
Effective absorbed power	P	2P
r/r_0 - beam radius at the level 86% of power	1	1.32
r_m/r_0 - the location of maximum of formula $r \cdot I(r)$	0.5	0.86
I_m/I_0 - intensity at the radius r_m/r_0	0.61	0.67

In particular, some evidences can be derived from the booklets of the firm TRUMPF. According to the TRUMPF data, two CO_2-lasers generating circularly polarized beams 3000 W (TEM_{00}) and 3800 W (TEM_{01*}) have the same technological possibilities in cutting mild steel, aluminum and stainless steel (Fig. 2). This allows estimating relative advantages of the R-TEM_{01*} mode in comparison with the circularly polarized TEM_{00} mode. The product of the cut depth by cutting velocity in the case of the R-TEM_{01*} mode increases by a factor of about 1.5.

The use of the R-TEM_{01*} mode gives some other possibilities. This mode occupies a larger volume of active media in comparison with the main mode. Hence the output power can be higher. The wider cut facilitates removing melted material and simplifies the organization of a gas jet in the kerf.

Laser beam propagating through the circular hollow metallic waveguide has losses connected with absorption of radiation on the waveguide walls at large angles of incidence. Here we deal with the same aspects of interaction of the beam with the metallic surface discussed in the case of laser cutting but pursue minimum losses. It is natural that azimuthal polarization is optimum for this purpose. According to the estimations made on the base of the Fresnel formulae, losses in a copper waveguide in the case of azimuthal polarization are lower by a factor of 2 than in the case of circular or linear polarization.

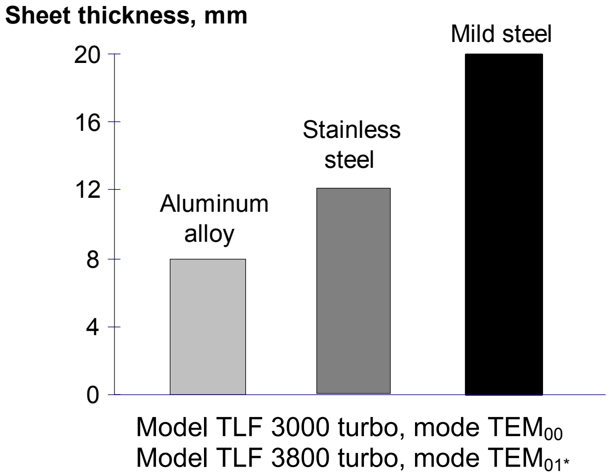


Fig.2. Maximum metal sheet thickness by cutting with two types of industrial lasers. Circular polarization. From advertisement of the firm Trumpf.

9. RESONANCE ABSORPTION OF RADIALLY POLARIZED RADIATION IN PLASMA

The problem of interaction of laser beams with plasma is of special interest. We consider the ASP beam energy transmission to plasma at inertial fusion.

One of the main processes in this case is resonance absorption by an inhomogeneous plasma target at which there is no limitation for temperature increasing in plasma [8-10]. The coefficient of resonance absorption depends upon angle between wave vector and electron-density gradient and also upon orientation of the electrical vector **E** relative to the plane of incidence. If the vector **E** is parallel to the plane of incidence the resonance absorption is maximum. If the vector **E** is perpendicular to the plane of incidence no resonance absorption occurs. In accordance with these features the use of linearly or circularly polarized radiation looks ineffective. In the case of the linearly polarized beam focused on a spherical target resonance absorption possesses an inhomogeneous distribution and doesn't occur in the plane perpendicular to the vector **E**. In the case of circularly polarized beam resonance absorption possesses axial symmetry but is time averaged, i.e. not optimum.

The whole surface of the target interacts with P-waves at focusing radially polarized beam on the target (Fig. 3). That is the radially polarized beam to provide maximum resonance absorption. For the radially polarized beam has a ring intensity distribution there is an optimum radius of the beam corresponding to the maximum absorption on the spherical surface. According to [8], we use the following expression for estimating the proposed effect:

$$f_w = \frac{I_{in} - I_{ref}}{I_{in}} = \frac{\tau^2}{2}\Phi(\tau), \qquad (13)$$

$$\tau = (2\pi z_c/\lambda)^{1/3}\sin\theta,\ \Phi(\tau) = 2.31\exp(-2\tau^3/3),$$

where I_{in}, I_{ref} are intensities of incident and reflected waves, λ is wave length, z_c is an electron density scale length by assuming linear electron density dependence of radius.

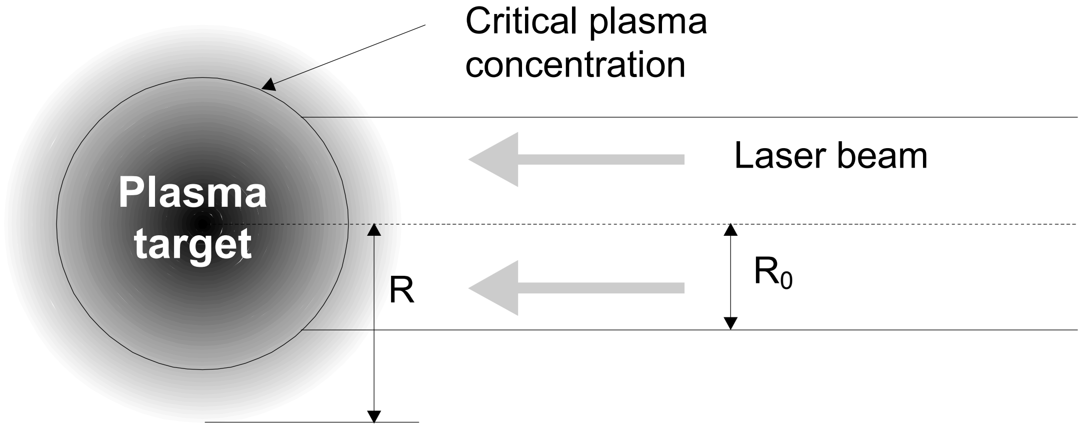


Fig.3. Interaction of the radially polarized beam with the spherical plasma target.

The parameter z_c characterizes the size of the outward plasma layer in which the electron density increases from zero to the critical value. The angle of incidence corresponding to maximum resonance absorption is defined by the expression

$$\sin\theta_{опт} \approx 0.8(\lambda/2\pi z_c)^{1/3}. \qquad (14)$$

An estimate for the resonance absorption on the spherical target may be made by the integral

$$W_{abs.}/W_0 = \frac{\int_0^{\pi/2} I(r,\theta,\varphi)F_w(\theta)\cos\theta\sin\theta d\theta}{\int_0^{\pi/2} I(r,\theta,\varphi)\cos\theta\sin\theta d\theta}. \qquad (15)$$

Here $I(r,\theta,\varphi)$ is an intensity distribution of the beam with radius R_0 on the target surface with radius R. The function $I(r,\theta,\varphi)$ has the form of the Laguerre Gauss intensity distribution. In the case of radial polarization $F_w(\theta)=f_w(\theta)$. As for linear or circular types of polarization, $F_w(\theta)=f_w(\theta)/2$.

In the experiments on inertial fusion λ=1.06 μm, $z_c \approx 1$ μm, therefore $\theta_{opt} \approx 20^o$ [9]. The plot of $W_{abs}/W_0(R_0)$ is presented in Fig. 4. The resonance absorption is maximum if a ring zone R_{max} corresponding to the maximum intensity over the beam cross section interacts with the target surface at the angle θ_{opt}, i.e. $R_{max}=R\sin\theta_{opt}$. If the size of the beam is optimized the resonance absorption doubles.

If the azimuthally polarized beam is focused on the spherical target there is no resonance absorption. The electrons in plasma oscillate in the wave electrical field along lines of equal density without generating electrostatic fields. Such type of polarization may be useful for investigating ponderomotive forces affecting electron density profile.

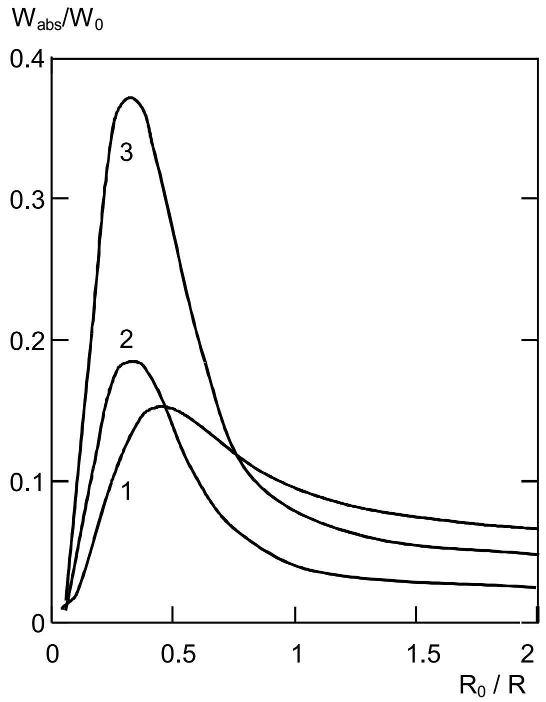


Fig.4. Resonance absorption on the spherical plasma target versus the radius of the laser beam (circularly polarized main mode (1), circularly polarized TEM01* mode (2), radially polarized TEM01* mode (3)).

10. CONCLUSION

The class of resonator modes with inhomogeneous polarization is deduced on the base of analysis of vector wave equation. It is shown that modes with axial symmetric polarization including radial and azimuthal have ring type intensity distribution.

There is a possibility of generating laser modes taking an intermediate place relative to polarization and intensity distribution between the linearly polarized mode TEM_{01} and the radially polarized mode TEM_{01*}.

The modes with inhomogeneous polarization can be selected by diffractive mirrors with polarization selectivity. These mirrors should be used as one of the laser resonator mirrors. For generating "pure" radially or azimuthally polarized modes diffractive mirrors with high polarization selectivity are necessary.

The radially polarized beam provides higher energy efficiency (the product of the depth of the cut by cutting velocity) for laser cutting metals than the circularly main mode does under the same conditions. The two limiting cases of resonance absorption on the spherical plasma target could be realized using axially polarized beams: the resonance absorption is maximum in the case of radial polarization and doesn't occur in the case of azimuthal polarization.

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