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Vector solution of the diffraction task using the Hertz vector

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A large class of diffraction problems can be solved on the basis of the Huygens principle. However, methods of solving diffraction problems based on this principle exhibit narrow boundaries of applicability. The goal of the present work is to offer a relatively simple physically based and mathematically strict "dipole wave" vector theory of nonparaxial diffraction of electromagnetic radiation which allows analytical solutions of typical diffraction problems. The suggested theory logically retains the wave approach used in the Kirchhoff method and does not exhibit strict limitations to applicability inherent in the Kirchhoff integral. The diffraction problem is solved by using the Hertz vector in the Kirchhoff integral instead of the field vector. The method efficiency is illustrated in several examples. Analytical solutions of diffraction base problems have been obtained for linearly polarized radiation on an infinite slit and on various-shaped holes at an arbitrary angle of incidence and polarization. It was shown the possibility of vector addition particular solutions to obtain diffraction patterns from several holes. The diffraction of radiation with azimuthal and radial directions of polarization on a ring slit is also considered. The main qualitative feature of the obtained solutions is the presence of "poles" one or two points of zero field in the diffraction pattern which are superimposed on the common system of light and dark fringes. The poles are located along electrical field vector directions. The vector analytical formulas describing the propagation of some laser beams in the free space have been obtained too. The solutions of the diffractive problems satisfy the Maxwell equations and the reciprocity principle.

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I. INTRODUCTION

The most common and strictest approach of solving diffraction problems is the solution of the vector Maxwell equations with corresponding boundary conditions. Due to its commonness, this approach is, in principle, applicable to any diffraction problem. Physical and mathematical methods of such an approach are described in many textbooks and monographs, for example, Refs. [1–6]. However, major mathematical difficulties with this method restrict the practical use of such solutions. So even formally strict solutions for diffraction on a round hole [7] cannot practically be used because of the bad convergence of the series in the form in which it is presented [3,8]. For solving concrete problems they usually apply physical simplifications, approximate calculations, and numerical methods.

A number of methods of roughing solving diffraction problems on the basis of the Huygens principle are known. The semiempirical approaches used in these methods impose strict limits on their applicability.

The Kirchhoff method for electromagnetic field description is the most famous. It uses the purely wave approach to solving diffraction problems. Derivation of the Kirchhoff integral is based on the wave equation and strict mathematical logic [2,4,9]. The Kirchhoff integral permits the diffraction field to be calculated using the field specified at some surface. Nevertheless, the area of the Kirchhoff integral applicability is rather narrow. The reason for these limitations is purely physical in nature and is located in the statement of the problem. The Kirchhoff integral is derived from two scalar wave equations, one of them being applied for the Green function and the other for a field propagating in free space. The scalar wave equation for the field does not contain any information on changes of field direction in space. The equation for the Green function gives a formal solution for the simplest spherical wave from a point source with uniform field direction. However, existence of such a wave is impossible due to the transverse nature of the electromagnetic field.

It is known that the Kirchhoff-Kottler integral is the generalization of the Kirchhoff method for the case of vector fields [2,4,9]. The common idea consists of the scalar Kirchhoff integral application to components of the field and in further vector adding of the obtained solutions. Characterizing this empirical approach, the authors of a classical monograph [9] point out that it does not have any physical interpretation and the solutions obtained on its basis do not satisfy the Maxwell equation $\operatorname{div}\mathbf{E}=0$. All the drawbacks of the scalar approach mentioned above are extended to the Kirchhoff-Kottler integral automatically. Therefore solutions on this base are approximate; they are correct in the narrow zone of diffraction pattern description. The authors of Ref. [10] considered in detail the famous paradox of classic paraxial approximation in the study of modes in spherical laser resonators. The solution for plane polarized mode has spherical wave front. This zero order approximation is contradictory to exact Maxwell equations. The first-order field is found to be a longitudinal field.

We shall also refer to the so-called electrodynamical formulation of the Huygens principle suggested by Kottler [9]. The initial point of this theory is the introduction of "equiva-

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lent surface currents" found from the field specified on the diaphragm.

Kottler suggested [11–13] the introduction of an additional contour (along the hole edge) integral into the solution. Kottler explained the necessity of this introduction by the presence of electric and magnetic charges distributed along the hole contour. Here we attempt to elucidate the field distribution in the hole and thus expand the method applicability. Nevertheless, this integral introduction cannot "compensate" for the shortcomings inherent in the Kirchhoff method itself, i.e., in the scalar integral and in vector generalization.

This paper is aimed at the creation of a physically justified and mathematically correct vector theory of diffraction that would use the logic of the Kirchhoff method and provide for the analytical solutions of several basic diffraction problems. The scope of this theory is broader than the applicability of the classical approach. The constraint on small solid angles, typical of the scalar approach, is not placed in this case. Although tasks with nontrivial boundary conditions at the aperture are not considered in the present paper, the proposed method using these conditions (unlike the scalar Kirchhoff integral) allows correct solutions of the diffraction task at small apertures. Some specific tasks on propagation of laser beams in space are solved. The obtained solutions are also valid for small radii of the initial field distribution leading to the large angle (nonparaxial) diffraction pattern.

A. Dipole-wave theory of diffraction

A mathematically correct and physically justified generalization of the Kirchhoff method for the diffraction vector theory is presented by the approach that employs not the field in the Kirchhoff method, but the polarization potential, or Hertz vector $\mathbf{Z}$ [2]. This approach is used, for example, in the antenna theory. As the fields found by the formula

$$\mathbf{E} = \nabla \times \nabla \times \mathbf{Z}, \quad \mathbf{H} = ik\nabla \times \mathbf{Z} \tag{1}$$

automatically satisfy the Maxwell equation $\operatorname{div}\mathbf{E}=0$, the inner contradiction inherent in the common Kirchhoff method is lacking in this approach.

The expression for the electromagnetic field of emitting dipole $\mathbf{p}=\mathbf{p}_0\exp(-i\omega t)$ is derived from the solution of the wave equation with the source in the form of dipole [2]

$$\mathbf{Z} = \mathbf{P}_0\frac{e^{-i\omega t+ikr}}{r}. \tag{2}$$

The polarization potential of the dipole wave is parallel to $\mathbf{p}_0$ and is transferred by a spherical wave. One of the two scalar equations used in the Kirchhoff method, i.e., the nonhomogeneous equation for the Green's function, can be quite exactly physically interpreted when written for Z. It describes the emission field of a real point source of electromagnetic radiation. Substituting Eq. (2) into Eq. (1), in the wave zone ($kr \gg 1$) we obtain a dipole wave $E_r=0$, $E_\theta=H_\varphi=-k^2Z\sin\theta$.

Noting that the dipole wave polarization potential retains its direction at different points of space, we can use the scalar equations applied to Z in the case of linearly polarized radiation falling on the aperture. Here the Kirchhoff scalar integral written for Z contains the vector information on the field of the diffraction pattern.

So, evaluation of the integral for $\mathbf{Z}$ is possible without any approximations due to vector function inhomogeneity, and the expressions for vector fields $\mathbf{E}$ and $\mathbf{H}$ can be obtained by simple differentiation. This approach is naturally extended for the general case of arbitrary distribution of the field direction on the aperture [14].

The basis for solving the vector problem of diffraction under linear polarization of incident radiation is provided by the Kirchhoff integral written for the Hertz vector:

$$Z(\mathbf{r}) = \int_{S'}[G(\mathbf{n}\cdot\nabla)Z_0 - Z_0(\mathbf{n}\cdot\nabla G)]dS'. \tag{3}$$

Here, $Z_0(\mathbf{r}')$ is the Hertz vector distribution on the given surface S', $Z(\mathbf{r})$ is the Hertz vector at the observation point, n is a unit normal to the given field surface on the aperture, and $G(\mathbf{r},\mathbf{r}')$ is the Green's function of the scalar wave equation.

For a widespread case of a plane polarized wave on the aperture $\mathbf{Z}_0(r')$ and $\mathbf{E}_0(r')$ on S' surface are related by a simple expression $\mathbf{E}_0(\mathbf{r}')=-k^2\mathbf{Z}_0(\mathbf{r}')$. Allowing for this, Eq. (3) can be represented as

$$\mathbf{Z}(\mathbf{r}) = -\mathbf{e}_0\frac{1}{k^2}\int_{S'}\left[G\frac{d}{dn}E_0 - E_0\frac{d}{dn}G\right]dS', \tag{4}$$

where $\mathbf{e}_0$ is a unit vector in the field direction and d/dn is a derivative in the direction $\mathbf{n}$, $d/dn=\mathbf{n}\cdot\nabla$.

The technology of diffraction field calculation is now reduced to evaluation of the integral in expression (4), writing of $\mathbf{Z}$ in vector form with the same unit vector as the $\mathbf{E}_0$ field, and to calculation of the field itself by formulas (1). The solution, obtained for the plane incident wave (4), permits the problem to be solved in the general case also.

We may conclude that the considered methodology of solving the diffraction problems presents some "physical" method of scalarization. In the general case of a three-dimensional vector field, the vector problem of diffraction must be solved for linearly polarized radiation with regard to each of the components. As stated above, this problem is reduced to the scalar integral by using the Hertz vector. These solutions satisfy the Maxwell equations, so the vectors of diffraction fields from all the components can be added.

We now discuss the applicability limitations for the considered method. A field with nondisturbed edges is employed on the aperture as a specified one. This causes a "physical" restriction for the typical size of the hole $a \gg \lambda$. The usual mathematical limitations in calculating diffraction integrals exist for the wave zone $kr \gg 1$ and for the hole size $ka \gg 1$, the latter being less strict as opposed to the limitation mentioned above. Here k is the wave vector, a is the typical aperture size, and r is the distance from the aperture edge to the observation point. In all the below problems, a more stringent approximation of $r \gg a$ is taken, which permits the analytical relationships to be derived. Combining the limitations for r and a, we can write in our calculations $r \gg a \gg \lambda$.

A key condition largely affecting the possibility of this method of practical applicability is the transformation of the

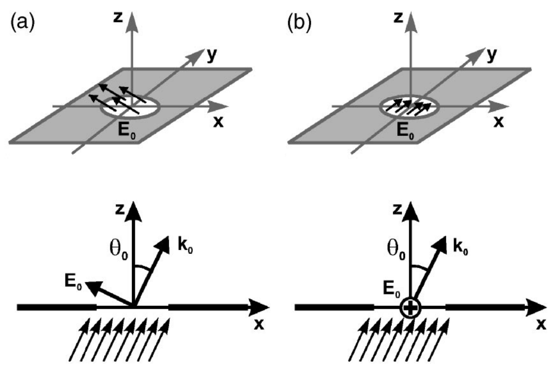

FIG. 1. Two possible directions of field on the aperture at nonzero angle of incidence: (a) field vector in plane of incidence; (b) field vector perpendicular to plane of incidence.

initial distribution of the field on the hole to the Hertz vector initial distribution. The general approach suitable for any given field distribution is hardly possible in this case. Below, the problems are solved which cover some particular cases: plane linearly polarized wave, and radially and azimuthally polarized radiation. A more general case can be also mentioned when the initial field shows linear polarization and the field amplitude distribution is such that everywhere $\nabla E_0 \perp \mathbf{E}_0$. Here we can use the decomposition of the field amplitude on the aperture into plane waves, and the field direction coinciding with E_0 can be ascribed to the scalar components of decomposition. The general solution of the problem is obtained by integrating with respect to the wave vector of the solution obtained for a plane wave.

Section V is concerned with a way of solving the problems of diffraction from several holes that employ the additivity of mathematical transformations used in the method under consideration. The propagation of laser beams in space is considered in the last section.

We offer several examples to illustrate the application of the suggested method. In all the cases the surface covering the aperture S' is taken to be flat in x-y coordinates, and the vector $\mathbf{n}$ in formulas (3) and (4) is directed along axis z. The plane x-z is the plane of radiation incidence. In diffraction from the slit or various form holes, two directions of field are possible (Fig. 1): in the plane of radiation incidence (a) and perpendicular to it (b). On this basis, solutions for any field direction can be obtained.

B. Diffraction of radiation from an infinite slit

Let us consider a plane linearly polarized wave incident to the slit (Fig. 1) in the plane x-z at the angle of θ_0 to the z axis:

$$\mathbf{E}_0 = \mathbf{e}_0 E_0 e^{i\mathbf{k}\cdot\mathbf{r}} = \mathbf{e}_0 E_0 e^{ikx' \sin\theta_0} e^{ikz' \cos\theta_0}.$$

The time factor $e^{-i\omega t}$ is omitted for brevity and $\mathbf{e}_0$ is a unit vector in the field direction. The vector $\mathbf{Z}$ (4) has the same direction in all the points that coincides with $\mathbf{E}_0$ direction. The scalar integral (4) is evaluated by a standard procedure. In our case, substituting E_0 and G, we can write it as

$$Z(r) = -\frac{1}{\mathbf{k}^2} E_0 \int_{S'} e^{ikx' \sin\theta_0} \times \left[G \frac{\partial}{\partial z'} e^{ikz' \cos\theta_0} - e^{ikz' \cos\theta_0} \frac{\partial}{\partial z'} G \right] dS'.$$

In the two-dimensional case considered here, the Green's function is expressed through the asymptotic approximation of the Hankel function of the first kind of the zeroth order:

$$G(k|\rho - \rho'|) \approx \frac{\sqrt{2}}{\sqrt{\pi}} e^{-i\pi/4} \frac{e^{ik|\rho-\rho'|}}{\sqrt{k|\boldsymbol{\rho} - \boldsymbol{\rho}'|}},$$

$$|\boldsymbol{\rho} - \boldsymbol{\rho}'| = \sqrt{(x-x')^2 + (z-z')^2},$$

where ρ is the radius vector directed to the point under consideration. Having performed the differentiation under the integral and setting $z'=0$, in the wave zone $k\rho \gg 1$ and for the case $\Delta x \ll \rho$ we obtain

$$\overline{Z} = -i \frac{\sqrt{2}}{\sqrt{\pi}} e^{-i\pi/4} \overline{S} \frac{e^{i\overline{\rho}}}{\sqrt{\overline{\rho}}} (\cos\theta_0 + \cos\theta) \frac{\sin\chi}{\chi},$$

$$\chi = \overline{\Delta x}(\sin\theta - \sin\theta_0). \tag{5}$$

Here we passed to the coordinates ρ and θ. The dimensionless parameters have also been entered:

$$\overline{Z} = Z\frac{k^2}{E_0}, \quad \overline{S} = 2k\Delta x, \quad \overline{\Delta x} = k\Delta x, \quad \overline{\rho} = k\rho.$$

This approximation is essentially different from that of Kirchhoff, as it does not impose any applicability limitations on the angle and contains the information on field direction at different points depending on the direction of the initial field. Expression (5) has been derived from the scalar integral and is valid for any initial field direction. It goes without saying that the expressions for the diffraction fields will differ according to the initial field direction, and hence $\mathbf{Z}$.

If the field E_0 is perpendicular to the plane of incidence ($E_0 \| y$), we have $\mathbf{Z} = Z(x,z)\mathbf{j}$ in the vector form. The calculation of the field from formula (1) was carried out in Cartesian coordinates.

The formulas for evaluating the magnetic and electric fields (1) are considerably simplified. The procedure of taking the derivatives is also rather simple if we consider their order of smallness from the different multipliers in formula (5). Because of the factor $k=2\pi/\lambda$, the derivative of the exponential with respect to the coordinate is maximum on the order of magnitude. The same factor appears in the derivative of χ, but this time with the coefficient $\Delta x/\rho$ that is small on the order of magnitude. That is why the second derivatives of Z with respect to the coordinates, which are needed in calculation of $\mathbf{E}$, have a very simple form

$$\frac{\partial}{\partial x} Z = -ik\frac{x}{\rho} Z, \quad \frac{\partial}{\partial z} Z = -ik\frac{z}{\rho} Z.$$

Then, allowing for Eq. (5) we rewrite the formula for $\mathbf{E}$ in the final form

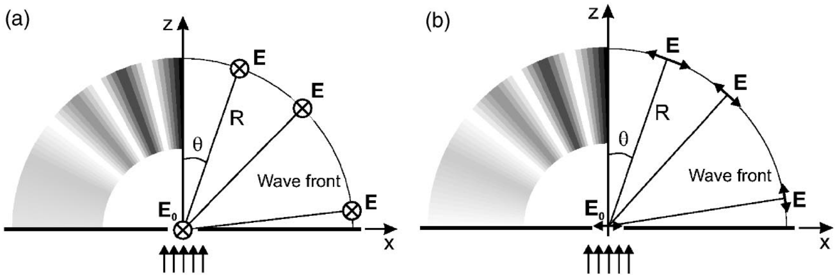

FIG. 2. Scheme, designations, and pattern of linearly polarized light diffraction from an infinite slit, synthesized by calculation. The diffraction field direction is illustrated by (a) and (b) at any field direction on the slit. The angle of incidence is zero.

$$\mathbf{E} = Z\mathbf{j} = -i\frac{\sqrt{2}}{\sqrt{\pi}}e^{-i\pi/4}E_0\bar{S}\frac{e^{i\bar{\rho}}}{\sqrt{\bar{\rho}}}(\cos\theta_0 + \cos\theta)\frac{\sin\chi}{\chi}\mathbf{j}, \quad (6)$$

$$\chi = \overline{\Delta x}(\sin\theta - \sin\theta_0), \quad \bar{S} = 2k\Delta x, \quad \overline{\Delta x} = k\Delta x, \quad \bar{\rho} = k\rho.$$

Expression (6), quite understandably, is identical to that obtained from the usual scalar Kirchhoff integral. Only in this unique case the usual method gives a solution satisfying the equation div $\mathbf{E}=0$, since the diffraction field has everywhere the same direction along y as the initial field, i.e., it only possesses one component, and this component is not dependent on y by virtue of the problem two-dimensionality.

Accordingly, for the field E_0 in the plane of incidence ($E_0 \perp y$), the polarization potential has the form $\mathbf{Z}=Z(x,z) \times(\mathbf{i}\cos\theta_0-\mathbf{k}\sin\theta_0)$, and formula (1) yields the following expression for the electric field $\mathbf{E}=E_0\cos(\theta-\theta_0)\overline{Z}\mathbf{e}_\theta$. So finally we obtain

$$\mathbf{E} = Z\cos(\theta-\theta_0)\mathbf{e}_\theta = -i\frac{\sqrt{2}}{\sqrt{\pi}}e^{-i\pi/4}E_0\bar{S}\frac{e^{i\bar{\rho}}}{\sqrt{\bar{\rho}}}(\cos\theta_0 + \cos\theta) \times\cos(\theta-\theta_0)\frac{\sin\chi}{\chi}\mathbf{e}_\theta, \quad (7)$$

$$\chi = \overline{\Delta x}(\sin\theta - \sin\theta_0), \quad \bar{S} = 2k\Delta x, \quad \overline{\Delta x} = k\Delta x,$$

$$\bar{\rho} = k\rho, \quad \mathbf{e}_\theta = \mathbf{i}\cos\theta - \mathbf{j}\sin\theta.$$

The direction of the diffracted field found from formula (7) is illustrated in Fig. 2. In any case, the formulas give a correct, perpendicular to the wave vector, direction of the field vector, and exhibit a slight difference in field amplitude for the two considered polarizations of the beam. With diffraction from a slit cut in a nontransparent screen, we cannot formally place strong emphasis on the qualitative differences in the diffraction pattern at large angles where $\cos(\theta-\theta_0)=0$.

In view of the above stated the approximation of a nondisturbed field on the aperture confines the hole size from below to, at least, several wavelengths. The intensity of the diffraction pattern therewith sharply falls when the polar angle is increased, and at large angles the field is practically zero with any polarization. However, if the slit is regarded as an element of wave front not restricted by a screen, the limitation on the slit size can be removed. In this case there is no need to account for the edge conditions in consideration of the field diffracted "along the screen" owing to its absence. The general propagation pattern for a wave with the specified distribution at $z=0$ will be found as the result of vector superposition of the fields diffracted from the front individual regions obtained by summing or integrating the corresponding expressions. The qualitative peculiarities of diffraction for any kind polarization may be essential here.

Figure 3 illustrates the distributions of field amplitude calculated by formulas (14) and (15) for an angle of incidence 30°. A characteristic feature of the case $\mathbf{E}_0 \perp y$ is the presence of an additional zero-field point at $\theta=\theta_0-\pi/2$ [see formula (7)]. This "pole" is located along the field direction at 60°.

Based on the obtained expressions (6) and (7), it is possible to revise the formula for the simplest diffraction grating formed by parallel slits cut in a nontransparent screen. It is known that the formula for the light that had passed through such a grating has two terms: a term for single slit diffraction

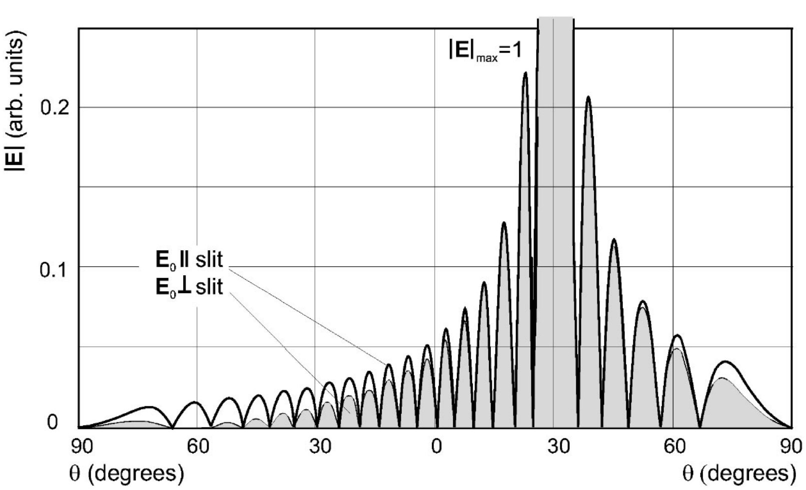

FIG. 3. Diffraction of a linearly polarized field on an infinite slit at two directions of polarization and with slit width 12λ. The angle of incidence is 30°.

and an expression related to the collective effect of diffraction from many slits. The formula for this grating will then take into account the polarization of radiation. If there is some distribution of the given field amplitude, the diffraction pattern can be calculated through the Green's function, by the solution.

In an arbitrary distribution of the specified field $\mathbf{E}_0 = E_{0x}\mathbf{i} + \mathbf{E}_{0y}\mathbf{j} + \mathbf{E}_{0z}\mathbf{k}$, it must be resolved into two components $\mathbf{E}_{0x-z} = E_{0x}\mathbf{i} + E_{0z}\mathbf{k}$ and $\mathbf{E}_{0y} = E_{0y}\mathbf{j}$. Making use of expressions for the infinitely small slit, we can write the general solution to the problem as

$$\mathbf{E} = -ik\frac{\sqrt{2}}{\sqrt{\pi}}e^{-i\pi/4}\frac{e^{ik\rho}}{\sqrt{k\rho}}(\cos\theta_0 + \cos\theta) \times \left[\mathbf{e}_\theta \cos(\theta_0 - \theta)\int_{x_1}^{x_2} E_{ox-z}(x_0)e^{ik(\sin\theta_0 - \sin\theta)x_0}dx_0 + \mathbf{e}_y\int_{x_1}^{x_2} E_{oy}(x_0)e^{ik(\sin\theta_0-\sin\theta)x_0}dx_0\right]. \quad (8)$$

Consider the example $\theta_0 = 0$, $E_{0y} = 0$, $E_{0x-z} = E_0\cos(\pi x_0/2a)$. The specified field is different from zero for $-a < x_0 < +a$. A simple calculation of the integral (8) gives the following formula for the diffraction field in this case as well:

$$\mathbf{E} = -ikaE_0\frac{4\sqrt{2}}{\sqrt{\pi^3}}e^{-i\pi/4}\frac{e^{ik\rho}}{\sqrt{k\rho}}(1+\cos\theta)\mathbf{e}_\theta\cos(\theta) \times \frac{\cos(ka\sin\theta)}{1 - 16\dfrac{a^2}{\lambda^2}\sin\theta^2}.$$

C. Diffraction of linearly polarized radiation from holes

The initial integral for calculating Z is written for the three-dimensional problem

$$Z = -iE_0\int_{S'} e^{ikx'\sin\theta_0}\frac{e^{ik|r-r'|}}{k|r-r'|}\left(\cos\theta_0 + \frac{z}{|r-r'|}\right)dS'.$$

The Green's function here has the form

$$G = \frac{e^{ik|r-r'|}}{k|r-r'|}.$$

The formulas for approximated calculations in terms of u are identical to the stated above decomposition with ρ substituting for r, though the formula for u has another form as integrating is now performed with respect to two coordinates:

$$u = \frac{xx' + yy'}{x^2 + y^2 + z^2} = \frac{xx' + yy'}{r^2} = u_x + u_y.$$

As a result, we arrive at the integral

$$Z = -\frac{i}{k}E_0(1 + \cos\theta\cos\theta_0)\frac{e^{ikr}}{r}I(\varphi,\theta), \quad (9)$$

$$I(\varphi,\theta) = \int_{S'} e^{ikx'\sin\theta_0}e^{-ikru}dS'.$$

While taking the integral in Cartesian coordinates, we must write

$$u = \sin\theta\cos\varphi\frac{x'}{r} + \sin\theta\sin\varphi\frac{y'}{r}.$$

The integral takes the form

$$I(\varphi,\theta) = \int_{S'}\int e^{ik(\sin\theta_0 - \sin\theta\cos\varphi)x'}e^{-ik\sin\theta\sin\varphi y'}dx'dy'. \quad (10)$$

On taking the integral in polar coordinates, the substitution must be made:

$$u = \frac{\rho'}{r}\sin\theta(\cos\varphi\cos\varphi' + \sin\varphi\sin\varphi') = \frac{\rho'}{r}\sin\theta\cos(\varphi - \varphi'), \quad x' = \rho'\cos\varphi'.$$

Then the following expression for the integral is derived:

$$I(\varphi,\theta) = \int_{S'}\int e^{ik\rho'[\sin\theta_0\cos\varphi' - \sin\theta\cos\varphi\cos\varphi' - \sin\theta\sin\varphi\sin\varphi']} \times d\rho' d\varphi'. \quad (11)$$

Thus, the Hertz vector is found from formula (9) with an integral similar to those of Eq. (10) or (11). Recall that the direction of the Hertz vector is the same as the field direction on the hole.

In calculation of the field by formula (1), consider two main directions of the field falling on the aperture: in the plane of incidence and perpendicular to it. Here, as well as in the case of a slit, the procedure of taking the derivatives in calculation of the rotor is not tedious if we take into account the smallness of the derivatives taken from the multipliers in formula (9).

The final expression for the field can be written as follows:

$$\mathbf{E} = -iE_0\frac{e^{i\bar{r}}}{\bar{r}}(\cos\theta_0 + \cos\theta)I(\varphi,\theta)\mathbf{q}(\varphi,\theta), \quad (12)$$

where $\bar{r} = kr$ and $I(\varphi,\theta)$ is found from formula (10) or (11) and specified by the hole shape, and the form of the vector function $q(\varphi,\theta)$ depends on the field direction on the hole (see Table I).

The solution for any direction of the field vector on the aperture can result from the vector addition of the solutions for $\mathbf{E}_0 \| y$ and $\mathbf{E}_0 \perp y$. In the case of a rectangular hole, the field is found from the formula

$$\mathbf{E} = -iE_0\bar{S}\frac{e^{i\bar{r}}}{\bar{r}}(\cos\theta_0 + \cos\theta)\frac{\sin\chi_a}{\chi_a}\frac{\sin\chi_b}{\chi_b}\mathbf{q}(\varphi,\theta), \quad (13)$$

$$\chi_a = \bar{a}(\sin\theta\cos\varphi - \sin\theta_0), \quad \chi_b = \bar{b}\sin\theta\sin\varphi,$$

TABLE I. The vector part of solutions of diffraction tasks for two directions of $\mathbf{E}_0$.

Field $\mathbf{E}_0$	Vector $\mathbf{Z}$	Vector $\mathbf{q}(\varphi,\theta)$
$\mathbf{E}_0 \parallel y$	$\mathbf{Z}=Z\cdot\mathbf{e}_y$	$\mathbf{e}_\theta \cos\theta \sin\varphi + \mathbf{e}_\varphi \cos\varphi$
$\mathbf{E}_0 \perp y$	$\mathbf{Z}=Z(\mathbf{e}_x \cos\theta_0 - \mathbf{e}_z \sin\theta_0)$	$\mathbf{e}_\theta(\sin\theta\sin\theta_0 + \cos\theta\cos\varphi\cos\theta_0) - \mathbf{e}_\varphi \sin\varphi\cos\theta_0$

$$\overline{S_0} = 4\overline{a}\overline{b}, \quad \overline{a} = ka, \quad \overline{b} = kb, \quad \overline{r} = kr,$$

where $2a$ and $2b$ are dimensions of rectangular hole along x and z axes, correspondingly.

For a rombolic hole obtaining the expression for the field

$$\mathbf{E} = -iE_0\overline{S}\frac{e^{i\overline{r}}}{\overline{r}}(\cos\theta_0 + \cos\theta)\left\{\frac{\sin\frac{\alpha+\beta}{2}}{\frac{\alpha+\beta}{2}}\frac{\sin\frac{\alpha-\beta}{2}}{\frac{\alpha-\beta}{2}}\right\}\mathbf{q}(\varphi,\theta), \tag{14}$$

$$\alpha = kc(\sin\theta_0 - \sin\theta\cos\varphi), \quad \beta = kd\sin\theta\sin\varphi,$$

$$\overline{S} = 2k^2cd.$$

$2c$ and $2d$ are diagonals of rombolic hole along x and z axes, correspondingly.

In the case of a circular hole, we should start with the integral (11). We deduce the following expressions for the Hertz vector $\mathbf{Z}$ and the electric field $\mathbf{E}$:

$$\mathbf{E} = -2iE_0\overline{S}\frac{e^{i\overline{r}}}{\overline{r}}(\cos\theta_0 + \cos\theta)\frac{J_1(\overline{r}M)}{\overline{r_0}M}\mathbf{q}(\varphi,\theta), \tag{15}$$

$$M = \sqrt{(\sin\theta_0 - \sin\theta\cos\varphi)^2 + (\sin\theta\sin\varphi)^2},$$

$$\overline{S_0} = \pi\overline{r_0^2}, \quad \overline{r} = kr, \quad \overline{r_0} = kr_0.$$

r is the radius of a hole.

Having prescribed $E_0 = E_0\delta(r' - r_0)$ we can deduce the formula for the Green's function that would permit the diffraction field to be calculated from the specified front with an arbitrary radial distribution of the field. We restrict ourselves to the case where the angle of incidence is zero.

If the given field is directed along the y axis, the expression for the Green's function for electric field takes the form

$$\mathbf{GE}_{(y)} = iE_0(k^2 2\pi r_0)(1+\cos\theta)\frac{e^{ik\mathbf{r}}}{kr}J_0(kr_0\sin\theta) \times [\mathbf{e}_\theta\cos\theta\sin\varphi + \mathbf{e}_\varphi\cos\varphi]. \tag{16}$$

When the given field is parallel to the x axis, we obtain

$$\mathbf{GE}_{(x)} = iE_0(k^2 2\pi r_0)(1+\cos\theta)\frac{e^{ik\mathbf{r}}}{kr}J_0(kr_0\sin\theta) \times [\mathbf{e}_\theta\cos\theta\cos\varphi - \mathbf{e}_\varphi\sin\varphi]. \tag{17}$$

With an arbitrary distribution of the specified field $\mathbf{E}_0 = E_{0x}\mathbf{i} + E_{0y}\mathbf{j}$ using expressions (16) and (17), the solution of the problem can be written as

$$\mathbf{E}_{(y)} = i(1+\cos\theta)\frac{e^{i\overline{r}}}{\overline{r}}[\mathbf{e}_\theta\cos\theta\sin\varphi + \mathbf{e}_\varphi\cos\varphi] \times \int_0^\infty 2\pi\overline{r_0}E_{0y}(\overline{r_0})J_0(\overline{r_0}\sin\theta)d\overline{r_0}, \tag{18}$$

$$\mathbf{E}_{(x)} = i(1+\cos\theta)\frac{e^{i\overline{r}}}{\overline{r}}[\mathbf{e}_\theta\cos\theta\cos\varphi - \mathbf{e}_\varphi\sin\varphi] \times \int_0^\infty 2\pi\overline{r_0}E_{0x}(\overline{r_0})J_0(\overline{r_0}\sin\theta)d\overline{r_0}. \tag{19}$$

The usual definitions $\overline{r} = kr$, $\overline{r_0} = kr_0$ are used here. The diffraction fields $E(y)$, $E(x)$ from the specified field components E_{0y} and E_{0x} must be added with regard to the components. The resulting diffraction field possesses two components, polar and azimuthal.

D. Discussion of calculation results for diffraction from holes

The results obtained feature the "poles" in the diffraction pattern, i.e., the points of zero field, located along the E_0 direction. For $\mathbf{E}_0 \parallel y$, two such points can be seen, their coordinates being $\varphi = \pm\pi/2$, $\theta = \pi/2$. With $\mathbf{E}_0 \perp y$ there exists one "pole" ($\varphi = \pi$, $\theta = \pi/2 - \theta_0$) in the observation hemisphere (Fig. 4).

The emergence of "poles" is assigned to the expression $\mathbf{q}(\theta,\varphi)$, and its form does not depend on the hole shape. Figure 5 depicts the distribution of field amplitude over the

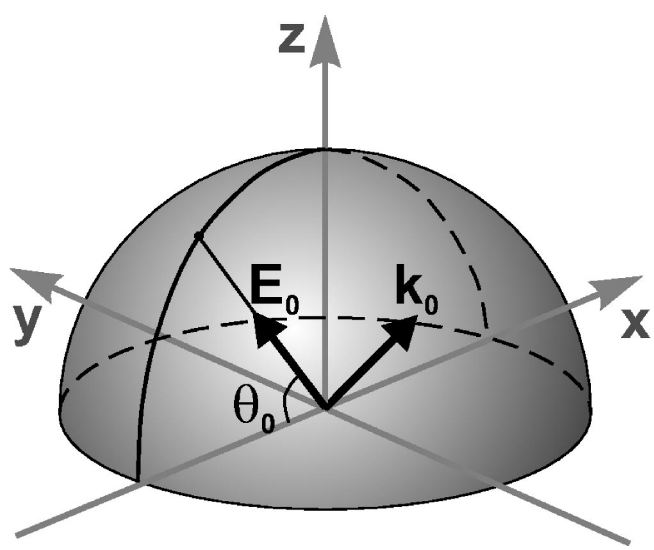

FIG. 4. Emergence of a "diffractive pole," a point of zero field, at oblique incidence of radiation on the slit, and with $\mathbf{E}_0$ located in the incidence plane.

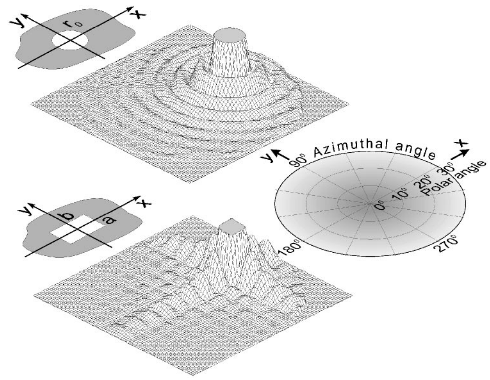

FIG. 5. Distribution of field amplitude under linearly polarized radiation diffraction from round and square holes in the coordinate system of polar and azimuthal angles. The round hole radius and the square side make 6λ. The angle of incidence is 15°. The main peak amplitude is normalized to a unit; the peaks in the figure are cut off at the level of 0.3.

hemisphere θ, φ in diffraction from the circular and square holes. The angle of incidence is 15°. The distinctions of diffraction patterns for different polarization directions, at rather large holes, are not visually distinguishable.

The formula for light diffraction from rectangular and circular holes, derived from the electrodynamical formulation of the Huygens principle, gives no description of the "poles" related to the direction of the E_0 field vector. The formulas derived in this paper are different from the known ones. They not only offer the qualitative features of diffraction pattern, but also provide for quantitative refinement of the field amplitude distribution over the diffraction pattern. Exact quantitative information is of great importance in this case, such as, for instance, the fact that the direction of maximum θ_m of the diffraction field is distinct from θ_0. The difference $\theta_0 - \theta_m$ depends on the angle of incidence and can reach several degrees.

The solutions of some diffraction problems obtained by the method of electrodynamical formulation of Huygens principle are not in agreement with the reciprocity principle. This discrepancy is typical of the problems where the surface currents are prescribed by the formula $\mathbf{j}=(c/2\pi)\mathbf{n}\times\mathbf{H}_0$. Concerning the outlined method, all the above solutions are consistent with the reciprocity principle. With $\varphi=0$; π, the angles θ and θ_0 are interchangeable.

E. Superposition of solutions at the diffraction from several holes

The vector solutions obtained for the diffraction from individual holes allow the diffraction pattern to be described in the presence of several holes. This can be performed through vector addition of the fields arriving at the specified point from different holes. The parameters (polarization, amplitude, relative phase shift, and angles of incidence) of the waves falling on the holes can be either identical or different.

Formally saying, allowing for the features of the employed mathematical operations, we can indicate that the operator $\mathbf{E}=L(\mathbf{E}_0)$ is capable of additivity. This approach can be used in resolving the initial field into the vector components, though bearing in mind that a linearly polarized initial field yields the diffraction field that is variously directed at different observation points. The diffraction from several holes can be considered in a similar way.

Primarily, the results must be generalized in the case of a random location of the hole in the screen relative to the origin of the coordinates. It gives an additional multiplier in the solution, describing phase distribution in the observation hemisphere. This phase distribution is associated with the asymmetrical location of the hole in the initial coordinate system. This multiplier has the form

$$F(\varphi,\theta,\theta_0,x_0,y_0) = e^{ik[x_0(\sin\theta_0 - \sin\theta\cos\varphi) - y_0\sin\theta\sin\varphi]}. \quad (20)$$

It does not depend on the hole shape and incident radiation polarization and is entered into all the finite formulas. So, for example, formula (15) for the circular hole displaced from the origin by x_0, y_0 takes the form

$$\mathbf{E} = -2iE_0 S\frac{e^{i\bar{r}}}{r}e^{i\omega t}e^{i\Psi}(\cos\theta_0 + \cos\theta)F(\varphi,\theta,\theta_0,x_0,y_0) \times \frac{J_1(\bar{r}_0 M)}{\bar{r}_0 M}\mathbf{q}(\varphi,\theta). \quad (21)$$

Here, for generality, we reproduced the time multiplier and entered a random phase shift Ψ that allows for the difference in phases of the beams falling on different holes. Generally, in addition to solutions of the diffraction field on the observation hemisphere not only being linearly polarized, they also have circular or elliptical polarization in different points.

If the holes are of the same size and symmetrical about the origin, the resultant formula is simplified, and the phase of the resulting field will be constant on the observation hemisphere. By way of example, consider the diffraction from six circular holes evenly spaced along the circle of R radius. The holes undergo the action of the plane wave. The general solution has the form

$$\mathbf{E} = -2iE_0 S\frac{e^{i\bar{r}}}{r}e^{i\omega t}(\cos\theta_0 + \cos\theta)(F_1+F_2+F_3) \times \frac{J_1(\bar{r}_0 M)}{\bar{r}_0 M}\mathbf{q}(\varphi,\theta). \quad (22)$$

F_1 corresponds to the first pair of holes located along the x axis:

$$F_1 = 2\cos\{kR(\sin\theta_0 - \sin\theta\cos\varphi)\},$$

F_2 and F_3 correspond to the other two pairs of holes

$$F_{2,3} = 2\cos\left\{k\left[\frac{R}{2}(\sin\theta_0 - \sin\theta\cos\varphi) \pm \frac{\sqrt{3}R}{2}\sin\theta\sin\varphi\right]\right\}.$$

Figure 6 presents the results of calculating the diffraction of a plane polarized wave from six circular holes, performed with formula (22).

For the two holes located along the x axis at R distance from the origin, the formula (21) includes

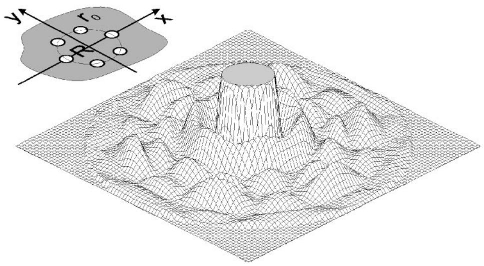

FIG. 6. Diffraction of plane polarized wave from six round holes. The field amplitude lies in the coordinates of azimuthal and polar angles (Fig. 5). The hole radius is $r_0=5\lambda$. The distance is $R=20\lambda$. Angle of incidence is zero. The main peak amplitude is normalized to a unit; the peak in the figure is cut off at the level of 0.1.

$$F(\varphi,\theta,\theta_0,x_0,y_0) = 2\cos\{kR(\sin\theta_0 - \sin\theta\cos\varphi)\}$$

for the case of zero phase shift and

$$F(\varphi,\theta,\theta_0,x_0,y_0) = -2i\sin\{kR(\sin\theta_0 - \sin\theta\cos\varphi)\},$$

when the field oscillations on the holes are out of phase. The diffraction field for these two cases is shown in Fig. 7.

The following figure (Fig. 8) illustrates the calculation of the field amplitude distribution for diffraction from two circular holes, when radiation falls on them at different angles $\pm\alpha$. From the two figures, for $\alpha=5°$ (top) and $\alpha=15°$ (bottom), how the diffraction pattern evolves with the increase of the incidence angle can be inferred. Analyzing the case schematically presented in Fig. 8, we let the planes of radiation falling on the holes coincide, the angles of incidence being equal but differing in sign.

If the vectors of the fields falling on the holes lie in the plane of radiation incidence (the field vectors being parallel), the diffraction fields at any point away from both the holes have the same direction, but differ in amplitude and phase. It can be convincingly indicated that the resulting diffraction field is linearly polarized at any point.

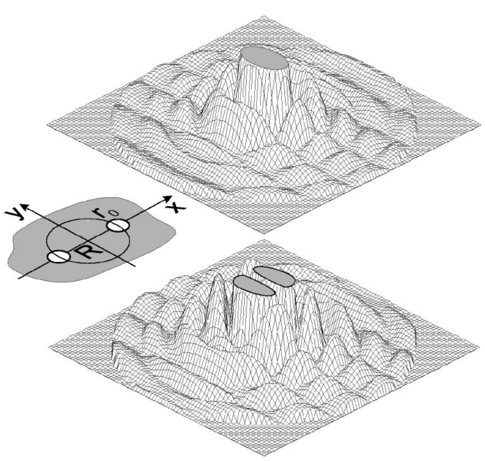

FIG. 7. Diffraction of plane polarized wave from two round holes. The field amplitude lies in the coordinates of azimuthal and polar angles (Fig. 5). The hole radius is $r_0=5\lambda$. The distance is $R=20\lambda$. Angle of incidence is zero. The main peak amplitude is normalized to a unit: the peaks in the figure are cut off at the level of 0.2.

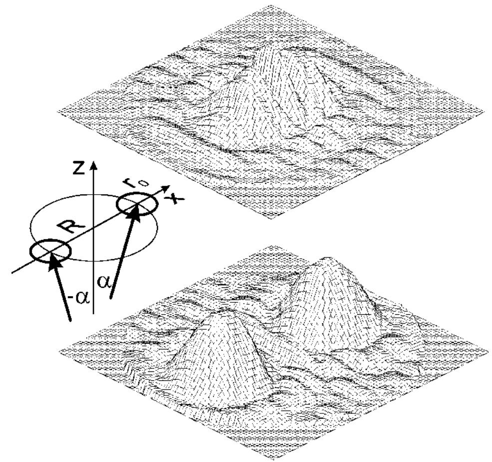

FIG. 8. Diffraction from two round holes. The field amplitude lies in the coordinates of azimuthal and polar angles (Fig. 5). The hole radius is $r_0=3\lambda$. The distance from the origin to hole center is $R=5\lambda$. The angle of incidence on the holes is $\pm\alpha$ ($\alpha=5°$ at the top, $\alpha=15°$ at the bottom).

If the field vectors falling on the holes are in the plane of radiation incidence, they are not parallel and form the angle 2α. In this case, the diffraction fields at any point away from the two holes are different not only in amplitude and phase, but in direction as well. The resulting fields diffracted from the two holes can be linearly or elliptically polarized at any point of the observation hemisphere.

F. Diffraction of azimuthally and radially polarized radiation from a circular slit

Modes with azimuthal and radial polarization are well known in the theory of waveguides and open resonators. Let us consider diffraction of light with this polarization from a circular slit. We shall restrict ourselves to the case of a zero angle of incidence $\theta_0=0$.

For an azimuthally polarized radiation assuming that within the circular aperture $\mathbf{Z}_0=Z_0e^{ikz}\mathbf{e}_\varphi$, it is readily shown by directly substituting into Eq. (6) that the relation $\mathbf{E}_0(r')=-k^2\mathbf{Z}_0(\mathbf{r}')$ holds here, too. The calculation of the Hertz vector is reduced to the integral

$$\mathbf{Z} = \frac{i}{k}E_0(1+\cos\theta)\frac{e^{ik\mathbf{r}}}{r}\int_{r'}\int_0^{2\pi}\mathbf{n}_\varphi e^{-ikr'\sin\theta\cos(\varphi-\varphi')}r'd\varphi'dr'.$$

Having written $\mathbf{n}_\varphi=-\mathbf{n}_x\sin\varphi+\mathbf{n}_y\cos\varphi$, we arrive at the scalar integrals

$$Z_x = -n_x\frac{i}{k}E_0(1+\cos\theta)\frac{e^{ikr}}{r}\times\int_{r'}\int_0^{2\pi}\sin\varphi'\, e^{-ikr'\sin\theta\cos(\varphi-\varphi')}r'd\varphi'dr',$$

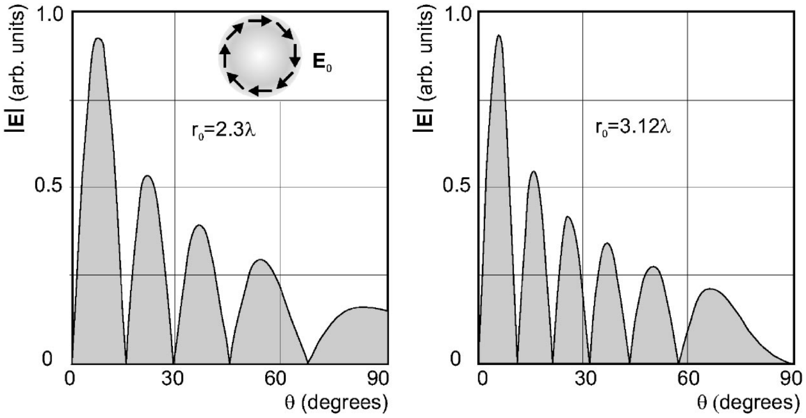

FIG. 9. Diffraction of an azimuthally polarized field on a narrow ring slit at two values of slit radius. Angle of incidence is zero.

$$Z_y = n_y \frac{i}{k} E_0 (1+\cos\theta) \frac{e^{ikr}}{r} \times \int_{r'} \int_0^{2\pi} \cos\varphi' \, e^{-ikr' \sin\theta \cos(\varphi-\varphi')} r' d\varphi' dr'.$$

Using further the known transformations [A1] and representing the initial field as $E_0 = E_0 \delta(r' - r_0)$, we obtain for the Hertz vector

$$\mathbf{GZ} = \mathbf{e}_\varphi E_0 2\pi r_0 (1+\cos\theta) \frac{e^{ik\mathbf{r}}}{kr} J_1(kr_0 \sin\theta).$$

In this case, the field is conveniently calculated by formula (1) in spherical coordinates. It takes the form

$$\mathbf{GE} = \mathbf{e}_\varphi k^2 E_0 2\pi r_0 (1+\cos\theta) \frac{e^{ikr}}{kr} J_1(kr_0 \sin\theta). \qquad (23)$$

Along the polar axis, for $\theta=0$, the field is zero. The magnitude of the field along the screen, for $\theta=\pi/2$, depends on the slit radius and may or may not be zero (Fig. 9). The general solution for $E_0(r_0)$ is determined by the integral

$$\mathbf{E} = \mathbf{e}_\varphi (1+\cos\theta) \frac{e^{i\bar{r}}}{\bar{r}} \int_0^\infty 2\pi \bar{r}_0 E_0(\bar{r}_0) J_1(\bar{r}_0 \sin\theta) d\bar{r}_0,$$

where, as usual, $\bar{r}=kr$, $\bar{r}_0 = kr_0$.

In the case of radially polarized radiation the solution for the Hertz vector has a similar form:

$$\mathbf{GZ} = E_0 (2\pi r_0) \left(1 + \frac{z}{\sqrt{\rho^2+z^2}}\right) \frac{e^{ik\sqrt{\rho^2+z^2}}}{k\sqrt{\rho^2+z^2}} J_1\left(kr_0 \frac{\rho}{\sqrt{\rho^2+z^2}}\right) e_\rho.$$

The above form has been adapted to calculation of the electric field (6) in the cylindrical coordinate system. The calculation results in the following formula:

$$\mathbf{GE} = k^2 E_0 2\pi r_0 \frac{e^{ikr}}{kr} (1+\cos\theta)\cos\theta J_1(\bar{r}_0 \sin\theta) \times \left\{ \cos\theta \mathbf{e}_\rho + \left[\frac{i}{\bar{r}\sin\theta} - \sin\theta\right] \mathbf{e}_z \right\}.$$

For convenience, we have here introduced natural notation in terms of $r=\sqrt{\rho^2+z^2}$ and the polar angle θ measured from the vertical axis.

It is convenient to decompose the obtained expression into two parts, a field in the meridional direction and a field along the z axis. The latter has a phase shift of $\pi/2$:

$$\mathbf{GE}_\theta = k^2 E_0 2\pi r_0 \frac{e^{i\bar{r}}}{\bar{r}} (1+\cos\theta)\cos\theta J_1(kr_0 \sin\theta) \mathbf{e}_\theta, \qquad (24)$$

$$\mathbf{GE}_z = ik^2 \frac{\bar{r}_0}{\bar{r}} E_0 2\pi r_0 \frac{e^{ikr}}{kr} (1+\cos\theta)\cos\theta \frac{J_1(kr_0\sin\theta)}{kr_0 \sin\theta} \mathbf{e}_z. \qquad (25)$$

The field distribution from formulas (24) and (25) is given in Fig. 10. The purely longitudinal component of the field (25) is small in magnitude (due to the factor r_0/r), but the maximum of the field is on the axis, where the meridional component (24) is zero. In addition, the longitudinal component is phase shifted by $\pi/2$ relative to the meridional field. The magnetic field possesses only an azimuthal component and "forms" a radial wave vector of the spherical wave together with the in-phase meridional component of the electric field. As for the field longitudinal component, it exhibits a $\pi/2$ phase shift with respect to the magnetic field, thus the time-averaged wave vector, related to this component, is zero.

The general solution for $E_0(r_0)$ is found from the formulas

$$\mathbf{E}_\theta = \mathbf{e}_\theta \frac{e^{i\bar{r}}}{\bar{r}} (1+\cos\theta)\cos\theta \int_0^\infty 2\pi \bar{r}_0 E_0(\bar{r}_0) J_1(\bar{r}_0 \sin\theta) d\bar{r}_0,$$

$$\mathbf{E}_z = i\mathbf{e}_z \frac{\bar{r}_0}{\bar{r}} \frac{e^{i\bar{r}}}{\bar{r}} (1+\cos\theta)\cos\theta \int_0^\infty 2\pi \bar{r}_0 E_0(\bar{r}_0) \frac{J_1(\bar{r}_0 \sin\theta)}{\bar{r}_0 \sin\theta} dr_0.$$

Undimensional notations $\bar{r}=kr$, $\bar{r}_0=kr_0$ are applied here. The above conclusions with regard to the meridional and longitudinal (along the z axis) components of the field are to full measure extended to the general case.

G. Propagation of laser beams in space

Now we illustrate on several examples the applicability of the Green function expressions to calculate of the formulas describing the propagation of some laser beams in space. The

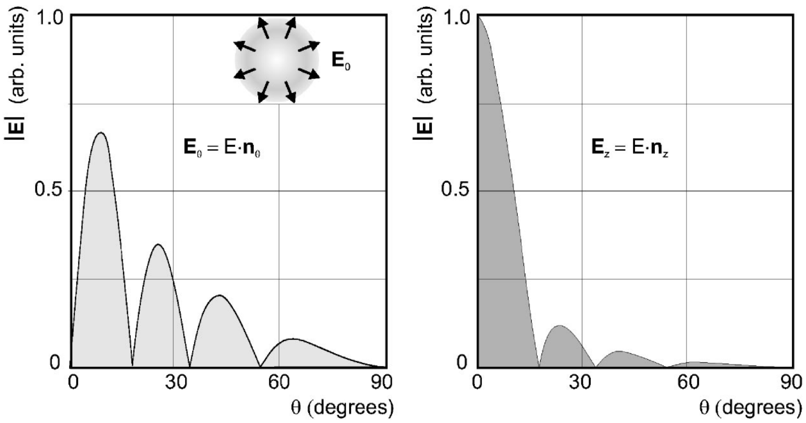

FIG. 10. Diffraction of a radially polarized field on a narrow ring slit ($r_0=2\lambda$). The left picture is the field component directed along meridian E_θ, the right picture is the field longitudinal component directed along z axis E_z. The longitudinal and meridian components of the field are displaced in phase by $\pi/2$. The relative scale of the two curves is not in agreement.

general method to solve these problems is as follows. It is suggested that the field distribution over the laser beam waist (wave front is flat and phase is constant) is known. Employing the expressions for the corresponding cases, we calculate the diffraction field at large distances (as against the typical size of the specified field) that governs the law beam propagation. In accordance with the general approach, the obtained solutions have a vector form, and they do not have the paraxiality restriction that is typical of the classical formulas of Gaussian beam propagation.

Consider the principal mode of Laguerre-Gaussian beams. Take for certainty that the specified field is linearly polarized and the field vector is directed along the y axis. It is required that the field distribution corresponding to this mode be substituted in formula (18):

$$E_a = \sqrt{\frac{2}{\pi}}\frac{1}{w_0}e^{-R_0^2}, \quad R_0 = r/w_0$$

and the integral using the known formula (A2) be taken as

$$\mathbf{E} = i\sqrt{\frac{2}{\pi}}\frac{1}{w_0}(1+\cos\theta)\frac{e^{i\bar{r}}}{\bar{r}}[\mathbf{e}_\theta \cos\theta \sin\varphi + \mathbf{e}_\varphi \cos\varphi] \times \int_0^\infty 2\pi \bar{r}_0 e^{-\bar{r}_0^2/k^2w_0^2} J_0(\bar{r}_0 \sin\theta) d\bar{r}_0.$$

The simple transformations yield the final expression giving a vector description of the Gaussian beam propagation in spherical coordinates:

$$\mathbf{E} = E_0(1+\cos\theta)\frac{e^{i\bar{r}}}{\bar{r}}[\mathbf{e}_\theta \cos\theta \sin\varphi + \mathbf{e}_\varphi \cos\varphi] e^{-(\bar{w}_0^2/4)\sin^2\theta}, \quad \bar{r} = kr,$$

$$\bar{w}_0 = kw_0.$$

The vector formulas for propagation of beams having azimuthal and radial polarization can be obtained in quite a similar way. The specified field distribution for the lower TEM_{01^*} mode is described in these cases by the formula

$$E_a = \sqrt{\frac{2}{\pi}}\frac{1}{w_0}(\sqrt{2}R_0)e^{-R_0^2}, \quad R_0 = r/w_0.$$

For the field having azimuthal polarization, we must apply formula (23), substitute the expression for the field into it, and perform an analytical taking of the integral. We obtain the final formula

$$\mathbf{E} = \mathbf{e}_\varphi E_0 \frac{e^{i\bar{r}}}{\bar{r}}(1+\cos\theta)\sin\theta e^{-(\bar{w}_0^2/4)\sin^2\theta}, \quad \bar{r} = kr, \quad \bar{w}_0 = kw_0.$$

Consider the specified field having radial polarization. Having performed a similar calculation employing formulas (24) and (25), we come to the following expressions for the field meridional and longitudinal components:

$$\mathbf{E}_\theta = \mathbf{e}_\theta E_0 \frac{e^{i\bar{r}}}{\bar{r}}(1+\cos\theta)\cos\theta \sin\theta e^{-(\bar{w}_0^2/4)\sin^2\theta},$$

$$\mathbf{E}_z = \mathbf{e}_z i E_0 \frac{e^{i\bar{r}}}{\bar{r}}\frac{\bar{w}_0}{\bar{r}}(1+\cos\theta)\cos\theta e^{-(\bar{w}_0^2/4)\sin^2\theta}.$$

Both in this case and in considering diffraction of radially polarized radiation through the circular slit, the field longitudinal component is small compared with the meridional field, but the maximum of this field is located at the axis where the field meridional component is zero. The longitudinal component of the field is also shifted by $\pi/2$ with relation to the meridional field.

The last example concerns the propagation of Bessel beams. Consider the case of the limited aperture of the initial field described by the Bessel zero-order function. A circle presents the aperture boundary. The given, but arbitrary number of zeros of the Bessel function, is brought into the aperture: $E_a \sim J_0(a_n r/r_n)$, $r \leqslant r_n$. Here r_n is the beam aperture radius including all zeros the Bessel function up to nth. The numerical factor a_n is consistent with the argument of the Bessel function at this zero $r=r_n$. Let the specified field have linear polarization and be directed along the y axis. As in the case of a Gaussian beam, we should apply formula (18), but substitute the Bessel function in the integral. Here also the integral is analytically taken by using the formula (A3). The calculation results in the formula

$$\mathbf{E} = E_0(1+\cos\theta)\frac{e^{i\bar{r}}}{\bar{r}}[\mathbf{e}_\theta \cos\theta \sin\varphi + \mathbf{e}_\varphi \cos\varphi]$$
$$\times\begin{cases} \dfrac{(a_n)^2}{(a_n)^2-(\bar{r}_n \sin\theta)^2} & J_0(\bar{r}_n\sin\theta), \\ \dfrac{a_n}{2}J_1(a_n) & \text{if } \bar{r}_n\sin\theta\to a_n, \end{cases}$$

$$\bar{r}=kr, \quad \bar{r}_n = kr_a.$$

At the large distances of $r \gg r_n$ the character of Bessel beam propagation along r differs not at all from the propagation of other beams. The features obtained in Ref. [15] for such beams do not take place in this case. The discussed general case involves the propagation of the waveguide laser principal mode in free space. This mode is described by the zero-order Bessel function and is limited by the tube walls in accordance with the first zero of Bessel function. In this case, $n=0$, $a_0=2.405$.

II. CONCLUSION

Consideration has been given to the method of solving diffraction tasks that is based on the Hertz vector application in the Kirchhoff integral. For this approach, there are no stringent limitations to applicability, inherent in the scalar Kirchhoff integral and its vector generalization written for the field. The method is physically sequential and mathematically correct, and does not exhibit inner contradictions; it presents a full vector approach and is technically simple. The search for a solution involves two stages. The first stage, calculation of the Hertz vector, is equivalent to taking the diffraction integral of the well-known class. The second stage consists in field calculation. Here, the operation of taking the curl is essentially simplified, if the order of summand smallness is allowed for taking the derivatives. The solution is dependent on polarization, but at any direction of linear polarization on the aperture it can be derived from the two base solutions for the vectors of the initial field being parallel and perpendicular to the plane of incidence. On plane wave incidence at a long distance from the aperture, the analytical vector expressions have been obtained for solving basic tasks of diffraction. The problems of linearly polarized radiation diffraction on an infinite slit, on holes of different forms, and on several holes have been considered. The problems of diffraction of azimuthally and radially polarized radiation on a ring slit were also studied. A qualitative feature of the obtained solutions is the presence of "poles," points of zero field which are superimposed on the common diffraction pattern consisting of light and dark fringes. The vector analytical formulas describing propagation of some laser beams in free space are obtained by the Green's function method. These solutions do not have paraxial approximation restrictions. The solutions satisfy the Maxwell equations and the reciprocity principle.

APPENDIX

Analytically taken integrals used for getting (searching) solutions of diffractive tasks.

$$\int_0^{2\pi} \sin(n\beta)\exp[it\cos(\beta-\gamma)]d\beta = 2\pi i^n J_n(t)\sin(n\gamma),$$

$$\int_0^{2\pi} \cos(n\beta)\exp[it\cos(\beta-\gamma)]d\beta = 2\pi i^n J_n(t)\cos(n\gamma), \tag{A1}$$

$$\int_0^\infty x^{\nu+1}e^{-\alpha x^2}J_\nu(\beta x)dx = \frac{\beta^\nu}{(2\alpha)^{\nu+1}}\exp\left(-\frac{\beta^2}{4\alpha}\right), \tag{A2}$$

$$\int_0^{r_n} rJ_0(\alpha r)J_0(\beta r)dr = \frac{\alpha r_n J_1(\alpha r_n)J_0(\beta r_n)}{\alpha^2-\beta^2}. \tag{A3}$$

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