Generation of inhomogeneously polarized laser beams by use of a Sagnac interferometer

V. G. Niziev, R. S. Chang, and A. V. Nesterov

A principal scheme for an external cavity technique for changing the polarization of a laser beam based on a modified Sagnac interferometer is proposed. The modified Sagnac interferometer includes standard optical components: a displacement polarizing beam splitter, an angle reflector, and a Dove prism. The radially polarized beams, obtained with the help of the developed scheme, allow the generation of a longitudinally polarized electric field by sharp focusing. The phase correction of radially polarized modes of higher orders leads to increasing the longitudinal field in the focus of the beam. © 2006 Optical Society of America

OCIS codes: 260.5430, 120.4640.

1. Introduction

Laser modes with inhomogeneous polarization have been known for quite some time. The classic works devoted to the theory of open resonators are based on the scalar wave equation.[1] Therefore the solutions describing field distribution over the cross section of a laser beam correspond to homogeneously polarized Gaussian modes, meaning that the state of polarization is the same in any point of the cross section of a laser beam. For round resonator mirrors, these solutions are the Laguerre–Gaussian modes. The authors point out the possibility of vector superposition of a couple of linearly polarized TEM_{01} modes to obtain radially and azimuthally polarized modes.[1]

There are many papers discussing the practical applications of inhomogeneously polarized modes. Radially polarized beams have been proposed for the laser cutting of metals[2] and azimuthal polarization for hole punching.[3] The radially polarized beam appears more efficient than a linearly polarized beam in experiments on laser heating plasma because of its higher resonance absorption.[4] The authors of Refs. 5 and 6 proved that a radially polarized beam can be focused more sharply than a linearly polarized beam. The longitudinal component of an electric field for a sharply focused radially polarized beam can be used for the acceleration of relativistic electrons.[7,8] These modes can be applied for trapping cold atoms,[9] and the helical modes are proposed in diagnostic and metrological systems.[10] This list of possible applications for modes with inhomogeneous polarization certainly does not stop here, but, it is too early to comment on the wide use of such modes because the methods for obtaining such modes are not yet reliable enough from a practical point of view.

Two main approaches to obtaining an inhomogeneously polarized mode (IPM) are internal-cavity and external-cavity techniques. One of the internal-cavity techniques uses a diffractive mirror[11,12] as one of resonator mirrors. This technique is preferable for lasers with high gain in the active media and low resonator quality. The long wavelength of radiation (CO_2 lasers) is an additional positive factor in this case because the period of diffractive structure is directly proportional to the wavelength.

An external-cavity technique can be used effectively for lasers with short wavelengths, low gain, and high resonator quality. The mode quality in such lasers is much higher than in high-power lasers. These high quality modes can therefore interact coherently outside the resonator. In turn, these external-cavity methods can be divided into two groups. We shall not discuss here the methods of transformation of the linearly polarized main mode TEM_{00} to a radially polarized mode. They use different special optical components for this: a polarization converter in the form of a quadrant half-wavelength

V. G. Niziev (niziev@yahoo.com) and R. S. Chang are with the National United University, Miao-Li, 36003, Taiwan. A. V. Nesterov is with the Institute on Laser and Information Technologies of the Russian Academy of Sciences, Shatura, Moscow Region, 140700, Russia.

Received 29 March 2006; revised 20 June 2006; accepted 2 July 2006; posted 3 July 2006 (Doc. ID 69458).

0003-6935/06/338393-07$15.00/0

plate,[5,13] a polarization converter on a base of liquid crystals,[14,15] and mode-forming holographic and birefringent elements.[16,17] These modes (initial TEM_{00} and desired TEM_{01*}) have a quite different field distribution, so the efficiency of such transformations is low despite the high efficiency of the local polarization transformation.

In this paper, attention is focused on the interferometric methods of reconstruction of a mode with inhomogeneous polarization, as in Refs. 7, 8, and 18. As with any method, this one has its advantages and problems, too. The main advantage of this method is its universality. It can be applied for any wavelength and for any type of IPM. The typical problems of interferometric methods, however, are well known. First, a high quality initial mode with good temporal and spatial coherence should be generated. The beam quality of a real laser is often far from the theoretical description of the empty resonator modes. Another problem is the alignment of the interferometer and its temporal stability. The field amplitude, mutual orientation of the mode patterns, and their polarization and phase correlation should be controlled. Taking into account the number of aligned optical components of the interferometer, this method appears rather complicated from a practical point of view. These problems are discussed in detail in Ref. 7. Naturally, the double interferometer scheme[8] has a dual problem concerning its alignment and stability. The purpose of this paper is to suggest a new procedure for constructing inhomogeneously polarized modes.

2. Principal Setup

A Melles–Griot He–Ne laser 05-LPB-670 is used with an output power of 6.0 mW. This laser has a tube with a sealed Brewster window at one end and a mirror at the other. The output mirror with a reflectivity of approximately 99% has a curvature radius of 45 cm. This laser can generate many transversal modes. The mode TEM_{01} with a controlled direction of plane polarization can be selected through the aperture. The mutual orientation of the mode pattern and the electric field vector can be changed by rotating the $\lambda/2$ phase shifter. A beam expander increases the beam diameter for convenience.

We used a Sagnac interferometer in our experiments. An attractive feature of the Sagnac interferometer is that both interfering beams sample the same optical path with the same elements, so distortions of the optics have a minimal effect on the sensitivity of the differential signal.[19,20] A simplified schematic of a Sagnac interferometer, which is based on standard optical components, is presented in Fig. 1(a).

The intensity distribution of two split beams can be mutually rotated by inserting a Dove prism (DP), as seen in Fig. 1(b). The spatial orientation of the intensity distributions of the beams in plane F just after the lateral displacement polarizing beam splitter is absolutely the same. Then each beam passes the DP and an angle reflector (AR) in different sequence and comes to the lateral displacement beam splitter,

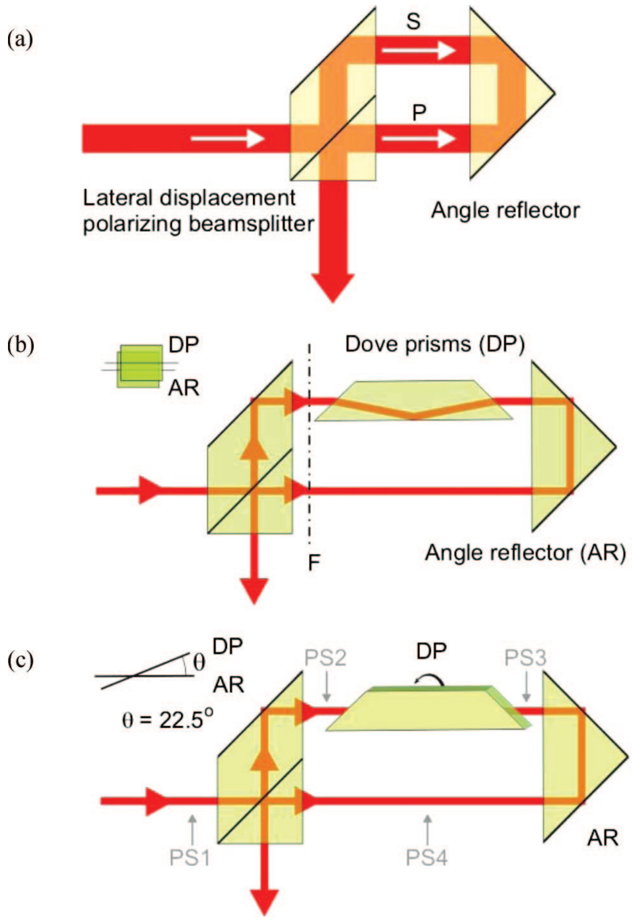

Fig. 1. (Color online) Schematic of a Sagnac interferometer. (a) Simplified schematic; P and S are the corresponding polarization of two beams. (b) Schematic with a Dove prism. (c) Modified Sagnac interferometer configuration to produce laser beams with inhomogeneous polarization.

which works now as a combiner. Both the DP and AR rotate the intensity distribution of a beam as a half-wavelength phase shifter (PS) rotates the electric field vector.[2,21] If both elements of the DP and AR have the same orientation in space, their common effect of rotating the beam intensity distribution will be zero. If the DP and AR possess a mutual orientation, as shown in Fig. 1(c) $\theta = 22.5°$, the intensity distribution of each of the two beams will be rotated around the beam axis in opposite directions at an angle of $2\theta = 45°$ so that the total mutual rotation angle is $4\theta = 90°$.

Four $\lambda/2$ phase shifters were installed in this schematic in the places indicated by the arrows. PS1 was used for correction of amplitudes of two beams after splitting. Correction was performed by rotating PS1 around the beam axis. The axes of the second and third PS are parallel to each other and oriented at an angle $\beta = 10.25°$ along the bisector of the angle θ between the DP and AR as shown in Fig. 2. These phase shifters are necessary to avoid changes of polarization made by the Dove prism. Without these two phase shifters, the Dove prism rotates the polarization of a beam passing through it because of the

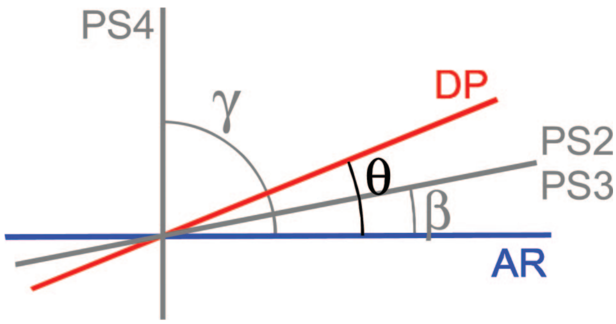

Fig. 2. (Color online) Graphical explanation concerning installation of half-wavelength phase shifters into experimental setup, where β = 10.25°, γ = 90°, and θ = 22.5°.

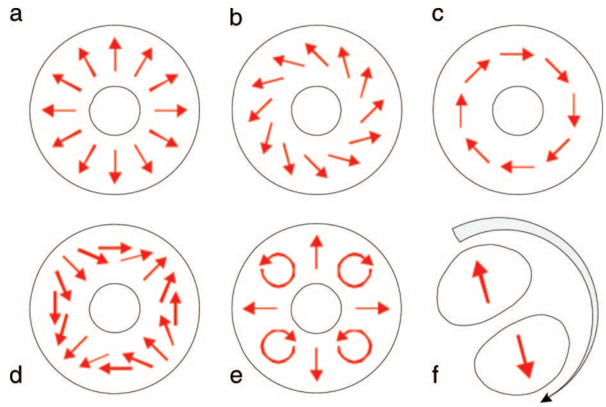

Fig. 4. (Color online) Examples of inhomogeneously polarized resonator modes. (a) Radially polarized mode R-TEM$_{01*}$. (b) The angle between **E** and radius is 45°. (c) Azimuthally polarized mode A-TEM$_{01*}$. (d) The mode with different directions of electric field. (e) The mode with different types of polarization. (f) Helical mode with plane polarization.

chosen mutual orientation of the incident beam polarization and the Dove prism (Fig. 1). The phase shifters PS2 and PS3 provide a means to avoid this. The beam polarization at the entrance and exit of the subsystem consisting of the three optical components (DP, PS2, and PS3) will be the same. The fourth PS was used for correcting a phase shift between two interfering beams on the beam combiner. The axis of this PS must be parallel (γ = 0°) or perpendicular (γ = 90°) to the AR edge. The correction of phase shift is performed by turning the PS around the line, which is perpendicular to the plane of drawing in Fig. 1. Figure 3 shows the corresponding setup mounting on the optical table.

3. Results and Discussion

The modes with radial and azimuthal directions of the electric field are only two representatives of the large family of inhomogeneously polarized modes that are solutions of the vector wave equation. Some of these modes are presented in Fig. 4.[4] The resulting distribution of parameters characterizing the state of polarization over the cross section of a laser beam depends on the mutual positions of these two modes, the amplitude and the orientation of the electric field, and a phase shift of their oscillations. A couple of Laguerre–Gaussian modes TEM$_{p1}$ (with $p \geq 0$ and the same indices) can coherently interact with each other creating, for example, a radially or azimuthally polarized mode of higher order, as indicated in Fig. 5.

A typical construction of the axially polarized mode TEM$_{01*}$ in a Sagnac interferometer consists of the following steps. First, an incident beam (TEM$_{01}$ mode) with plane polarization is split. Then the two beams are mutually turned by 90° and come to the beam combiner with perpendicular polarizations.

The experimental results of the typical diagnostics of radially and azimuthally polarized beams are presented in Fig. 6. The reconstructed beam from a Sagnac interferometer has a ring-type distribution of intensity [Fig. 6(a)]. This beam passes through a polarizer–analyzer, and the resulting picture is fixed on the screen. Figure 6(b) shows the experimental pictures for a mode indicated in Fig. 4(e). The single difference between the mode in Fig. 4(e) and the radially polarized mode in Fig. 5(a) consists of a phase shift between two mode patterns combined at the exit of the interferometer. The diagnostically obtained pictures for a radially polarized beam are pre-

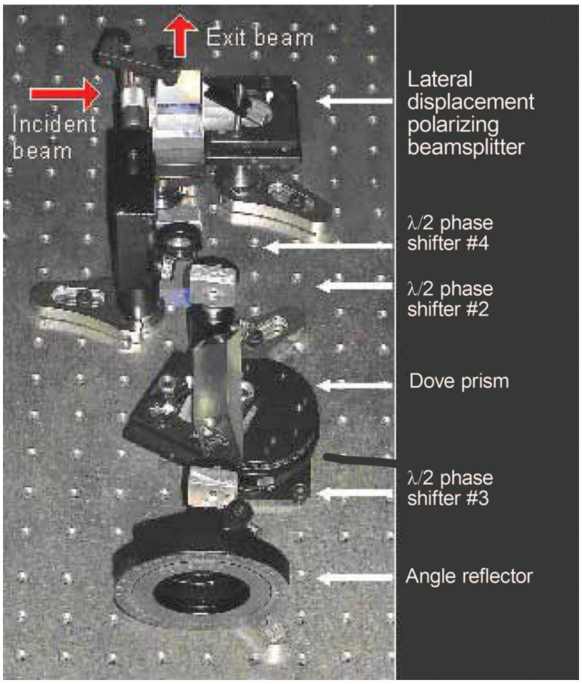

Fig. 3. (Color online) Modernized Sagnac interferometer mounting on an optical table.

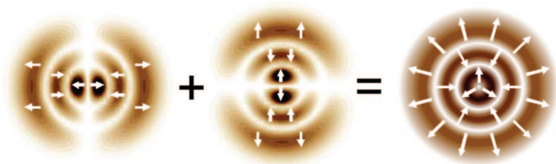

Fig. 5. (Color online) Radially polarized mode of higher order (R-TEM$_{21*}$).

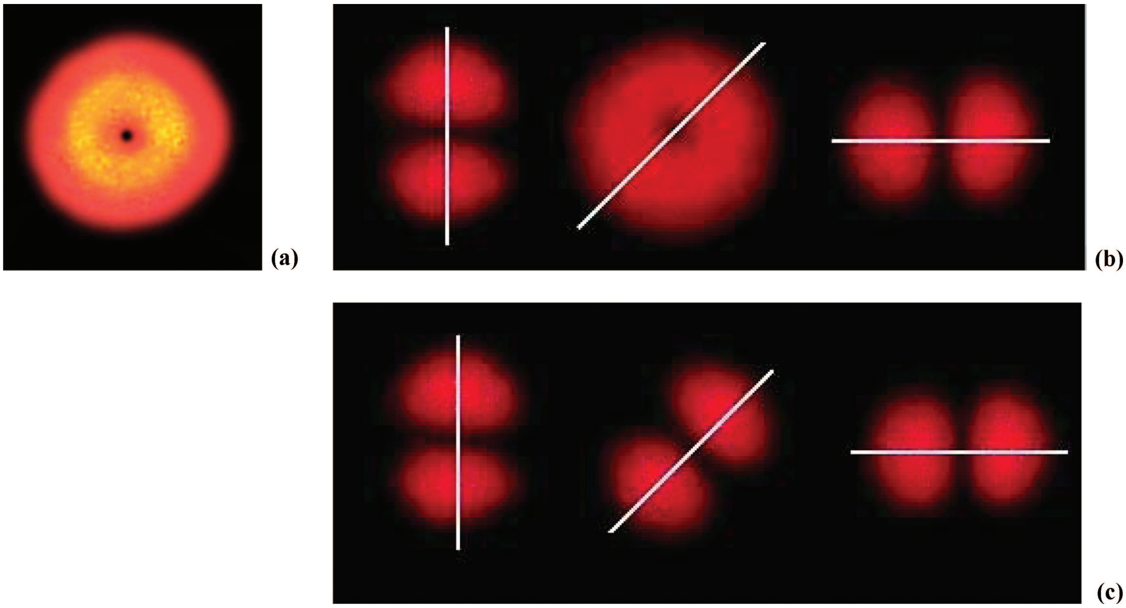

Fig. 6. (Color online) Experimentally obtained pictures of the modes with axially symmetric polarization. (a) Intensity distribution in the cross section of the laser beam. (b) Diagnostics of the mode with inhomogeneous polarization indicated in Fig. 5(e). The intensity distribution is just after the polarizer–analyzer. The white line is the axis of the polarizer. (c) The diagnostics of the radially polarized beam indicated in Fig. 5(a). The white line is the axis of the polarizer. The mode pattern rotates with the rotation of a polarizer around the beam axis.

sented in Fig. 6(c). The spots on the screen look like a pure TEM_{01} mode and rotate synchronically with the rotation of the polarizer axis. In the case of an azimuthally polarized beam, the polarizer axis and the mode pattern are mutually turned at 90° compared to the radially polarized beam. At any orientation of the polarizer axis, two spots will have the same position relative to this axis.

The suggested scheme [Fig. 1(c)] based on a Sagnac interferometer is simple and inherently stable. The amplitudes of two interfering beams and their mutual phase shift are controlled and variable. The electric field vector of the linearly polarized incident beam must be at 45° to the plane of the interferometer, giving the same amplitude of the two beams after being split. The two spots of TEM_{01} mode in the incident beam can have any orientation around the beam axis. Independent of that, the field distribution at the exit will be axially symmetric. If the phase shift between the two beams at the combiner is zero, the possible states of polarization are shown in Figs. 4(a)–4(c). The resulting field distribution is determined by the mutual position of the mode pattern and field direction of the incident beam. The next two polarization distributions, Figs. 4(d) and 4(e), can be realized by choosing appropriate parameters of the incident beam and phase shift between the interfering beams. To obtain a helical mode, Fig. 4(f), the beam splitter (combiner) should be changed: A polarizing beam splitter must be replaced with a nonpolarizing one.

An interesting additional possibility of such a scheme should also be mentioned. Because this interferometer has one path exit, it can be used as an interferometric rear mirror of the laser resonator (Fig. 7) with a controlled state of polarization of an inhomogeneously polarized beam (radial, azimuthal and others). The orientation of the electric vector determined by the polarizer (in this part of the sche-

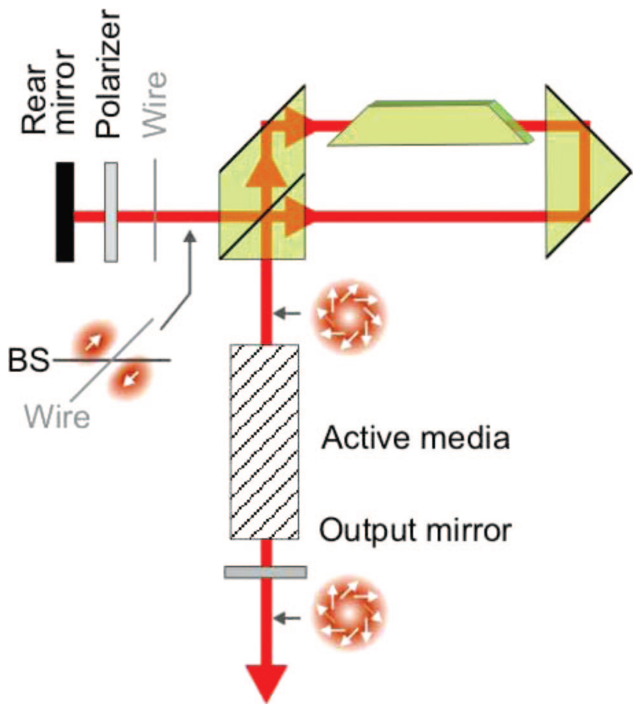

Fig. 7. (Color online) Schematic of use of a Sagnac interferometer in a laser resonator as a rear mirror to generate inhomogeneously polarized modes.

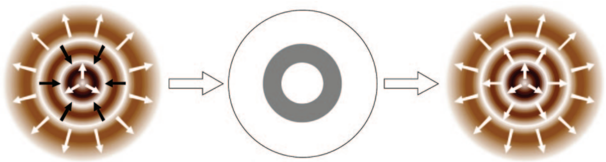

Fig. 8. (Color online) Phase correction of the high-order radially polarized mode.

matic) should always be at 45° to the polarizing beam splitter. The orientation of the mode here can be changed by rotating a wire around the beam axis. It leads to different states of polarization of the exit beam, from radial to azimuthal. This idea has not been tested yet. To do so, one would need a laser with a higher gain than that possessed by the He–Ne laser.

One of the most interesting phenomena connected with inhomogeneously polarized laser beams is a longitudinal component of the electric field in the focal spot of the lens. It is interesting from the physical point of view because the energy associated with this component is not transmitted. This component of the electric field (especially in the case of radially polarized beams) can be used in some applications discussed in Section 1. Therefore methods of increasing the longitudinal component of the electric field are of interest. The common idea of selectively changing phases of separated spots inside the high-order mode[22] can be applied in this case too.

This plan includes several steps:

1. Generation of a high-order mode from the class TEM_{p1} by using a special resonator mirror with a corresponding mask on its surface[22] or other methods.[23]
2. Transformation of this mode to a radially polarized mode of high-order R-TEM_{p1*} in the Sagnac interferometer as described in this paper.
3. Correction of the phase distribution of the R-TEM_{p1*} by a local phase corrector to make a uniform phase over the cross section of the laser beam, as seen in Fig. 8.
4. Focusing of the corrected beam to obtain the maximum amplitude of the longitudinal component of the electric field in the focal spot of a lens.

The classical solution for the homogeneously polarized mode cannot be used directly for calculation of the longitudinal component of field just following the pictures such as those in Fig. 5 for the radially polarized mode. There are two reasons for that. Classical solutions neglect the longitudinal component, and they are in contradiction with the Maxwell equation $\nabla\mathbf{E} = 0$. A correct way for such calculations was pointed out in Ref. 4. It was shown there that the spatial part of the classical solution for the Laguerre–Gaussian modes (obtained for a homogeneously polarized mode) strictly corresponds to the paraxial approximation of the solution for the azimuthally polarized field (inhomogeneously polarized mode). The radial and longitudinal components of the other field must be calculated through the Maxwell equations. This description does not have any inner contradictions and corresponds to the Maxwell equation $\nabla\mathbf{E} = 0$.

The method of calculating the longitudinal field for a high-order radially polarized mode is presented in Refs. 4 and 24. According to this method, the longitudinal components of the electric vector can be expressed as

$$E_{p,1}^{z}(r, z) = \frac{1}{r}\frac{\partial\left(rH_{p,1}^{\varphi}(r, z)\right)}{\partial r}, \tag{1}$$

where H_{p1}^{φ} is the single component of the magnetic field with azimuthal direction. Using in Eq. (1) the classic formula for Laguerre–Gaussian beams,[25]

$$H_{p,1}^{\varphi} = \sqrt{\frac{2p!}{\pi(p+1)!}}\frac{1}{w}\left(\sqrt{2}R\right)L_p^{1}(2R^2)\exp(-R^2)\exp(i\theta),$$

$$\theta = 2\arctan Z - 2Z\frac{z_0^{2}}{w_0^{2}} - ZR^2;$$

$$R = r/w,\ w^2 = w_0^{2}(1+Z^2);\quad Z = z/z_0;\quad z_0 = \frac{\pi w_0^{2}}{\lambda};$$

$$L_p^{1}(\mathrm{x}) = \sum_{m=0}^{p}(-1)^m\frac{(p+1)!}{(p-m)!\,(m+1)!\,m!}x^m,$$

gives the expression for the longitudinal component of the electric field:

$$E_{p1}^{z}(r, z) = -2\mathrm{i}\frac{1}{\pi}\sqrt{\frac{p!}{\pi(p+1)!}}\frac{\lambda}{w}\frac{1}{w}\Big\{(1-R^2-iR^2Z) \times L_p^{1}(2R^2) + \left[x\frac{d}{\mathrm{dx}}L_p^{1}(x)\right]_{x=2R^2}\Big\} \times \exp(-R^2)\exp(\mathrm{i}\theta). \tag{2}$$

The transformation of Eq. (2) for the cross section in the waist ($z = 0$) according to the expression

$$x\frac{\mathrm{d}}{\mathrm{dx}}L_p^{1}(x) = \left[pL_p^{1}(x) - (p+1)L_{p-1}^{1}(x)\right]$$

which leads to the form convenient for numerical calculation:

$$E_{p,1}^{z}(r) = -2\mathrm{i}\frac{1}{\pi}\sqrt{\frac{p!}{\pi(p+1)}}\frac{\lambda}{w_0}\frac{1}{w_0}\Big\{(1+p-R_0^{2}) \times L_p^{1}(2R_0^{2}) - \left[(p+1)L_{p-1}^{1}(2R_0^{2})\right]\Big\} \times \exp(-R_0^{2}), \tag{3}$$

where $R_0 = r/w_0$, and w_0 is the radius in the waist. The radial distributions of the longitudinal compo-

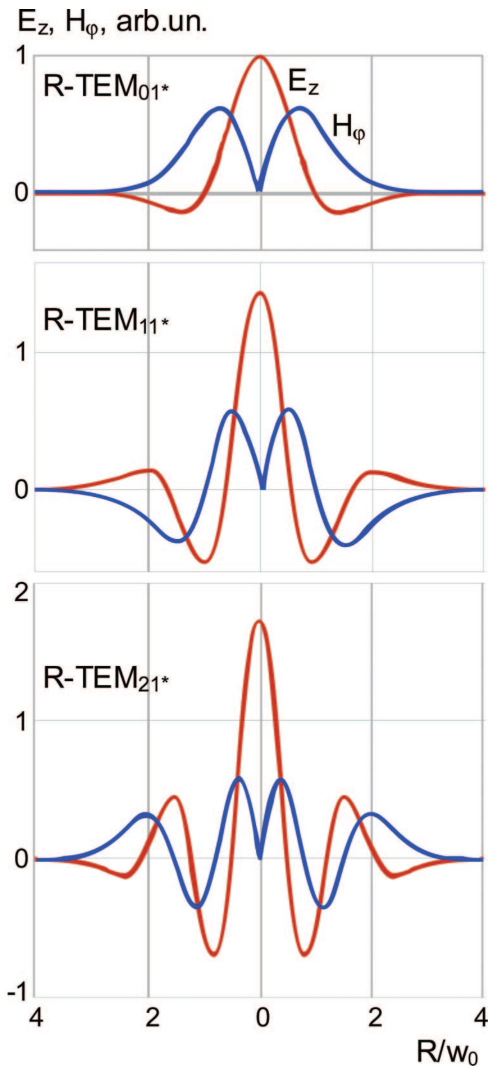

Fig. 9. (Color online) Calculated distributions of longitudinal components of electric field E_z and azimuthally directed magnetic field H_φ in the waist for the radially polarized modes of different orders.

nent of the electric field $E_{p1}{}^{z}(r)$ for the Laguerre–Gaussian modes of different orders ($p = 0, 1, 2$) calculated according to formula (3) are presented in Fig. 9. The longitudinal component of the electric field at the beam axis increases with the increasing order of a radially polarized mode. After phase correction (Fig. 8), the maximum value of the longitudinal field will be obtained in the lens focus.

4. Conclusion

The couple of Laguerre–Gaussian modes TEM_{p1} (with $p \geq 0$ and the same indices) can coherently interact with each other creating inhomogeneously polarized modes. A principal scheme for an external cavity technique for changing the polarization of the laser beam based on a modified Sagnac interferometer is proposed.

The modified Sagnac interferometer includes standard optical components: a displacement polarizing beam splitter, an angle reflector, and a Dove prism. A linearly polarized laser mode TEM_{p1}, coming into the Sagnac interferometer, is split into two beams after passing the lateral displacement polarizing beam splitter. These beams, having perpendicular polarization directions, propagate in opposite directions along the same optical path. The intensity distribution of two split beams will be rotated around the beam axis in opposite directions using the Dove prism. The electric field vector of the linearly polarized incident beam must be directed at 45° to the plane of the interferometer, giving the same amplitude of the two beams after being split. The two spots of TEM_{p1} mode in the incident beam can have any orientation around the beam axis. Independent of that, the field distribution at the exit will be axially symmetric.

The new method based on a Sagnac interferometer is simple and inherently stable. It is applicable to the highly efficient generation of different types of inhomogeneously polarized modes with the same setup.

The radially polarized beams, obtained with the help of the developed scheme, allow the generation of a longitudinally polarized electric field by sharp focusing. The phase correction of radially polarized modes of higher orders leads to an increase of the longitudinal field in the lens focus.

The authors thank Jean-Francois Bisson for useful discussions.

References

1. W. Koechner, *Solid-State Laser Engineering* (Springer, 1988), p. 172.
2. V. G. Niziev and A. V. Nesterov, "Influence of beam polarization on laser cutting efficiency," J. Phys. D **32,** 1455–1461 (1999).
3. M. Meier, H. Glur, E. Wyss, Th. Feurer, and V. Romano, "Laser microhole drilling using *Q*-switched radially and tangentially polarized beams," in Proc. of SPIE **6053,** 313–318 (2005).
4. A. V. Nesterov and V. G. Niziev, "Laser beams with axially symmetric polarization," J. Phys. D **33,** 1817–1822 (2000).
5. R. Dorn, S. Quabis, and G. Leuchs, "Sharper focus for a radially polarized light beam," Phys. Rev. Lett. **91,** 233901 (2003).
6. C. J. R. Sheppard and A. Choudhury, "Annular pupils, radial polarization, and superresolution," Appl. Opt. **43,** 4322–4327 (2004).
7. S. C. Tidwell, D. H. Ford, and W. D. Kimura, "Generating radially polarized beams interferometrically," Appl. Opt. **29,** 2234–2239 (1990).
8. S. C. Tidwell, G. H. Kim, and W. D. Kimura, "Efficient radially polarized laser beam generation with a double interferometer," Appl. Opt. **32,** 5222–5229 (1993).
9. A. V. Bezverbny, V. G. Niziev, and A. M. Tumaikin, "Dipole traps for neutral atoms formed by nonuniformly polarized laguerre modes," Quantum Electron. **34,** 685–689 (2004).
10. T. I. Arsenyan, N. N. Fedotov, L. S. Kornienko, P. V. Korolenko, E. A. Kulyagina, and G. V. Petrova, "Laser beam with helical wavefront dislocations and their applications in the diagnostical and metrological systems," in *Fifth International Conference on Industrial Lasers and Laser Applications '95,* V. Panchenko and V. Golubev, eds., Proc. SPIE **2713,** 453–460 (1995).
11. A. V. Nesterov, V. G. Niziev, and V. P. Yakunin, "Generation of high-power radially polarized beam," J. Phys. D **32,** 2871–2875 (1999).
12. T. Moser, M. A. Ahmed, F. Pigeon, O. Parriaux, E. Wyss, and Th. Graf, "Generation of radially polarized beams in Nd: YAG

lasers with polarization selective mirrors," Laser Phys. Lett. **1,** 234–236 (2004).
13. S. Quabis, R. Dorn, and G. Leuchs. "Generation of radially polarized doughnut mode of high quality," Appl. Phys. B **81,** 597–600 (2005).
14. G. Miyaji, N. Miyanaga, K. Tsubakimoto, K. Sueda, and K. Ohbayashi, "Intense longitudinal electric fields generated from transverse electromagnetic waves," Appl. Phys. Lett. **84,** 3855–3857 (2004).
15. G. Miyaji, K. Ohbayashi, K. Sueda, K. Tsubakimoto, and N. Miyanaga, "Generation of vector beams with axially-symmetric polarization," Rev. Laser Eng. **32,** 259–264 (2004).
16. E. G. Churin, J. Hoβfeld, and T. Tschudi, "Polarization configurations with singular point formed by computer generated holograms," Opt. Commun. **99,** 13–17 (1993).
17. Z. Bomzon, G. Biener, V. Kleiner, and E. Hasman, "Radially and azimuthally polarized beams generated by space-variant dielectric subwavelength gratings," Opt. Lett. **27,** 285–287 (2002).
18. N. Passilly, R. Denis, K. Ait-Ameur, F. Treussart, R. Hiorle, and J. F. Roch, "Simple interferometric technique for generation of a radially polarized light beam," J. Opt. Soc. Am. A **22,** 984–991 (2005).
19. P. T. Beyersdorf, M. M. Fejer, and K. L. Byer, "Polarization Sagnac interferometer with postmodulation for gravitational-wave detection," Opt. Lett. **24,** 1112–1114 (1999).
20. J. Hwang, M. M. Fejer, and W. E. Moerner, "Scanning interferometric microscopy for the detection of ultrasmall phase shifts in condensed matter," Phys. Rev. A **73,** 021802(R) (2006).
21. Q. Zhan and J. R. Leger, "Focus shaping using cylindrical vector beams," Opt. Express **10,** 324–331 (2002).
22. M. G. Galushkin, P. V. Korolenko, V. G. Makarov, A. T. Polosko, and V. P. Yakunin, "Phase correction of radiation emitted by a powerful industrial laser with higher mode selection," Quantum Electron. **32,** 547–552 (2002).
23. R. Oron, Y. Danziger, N. Davidson, A. A. Friesem, and E. Hasman, "Discontinuous phase elements for transverse mode selection in laser resonators," Appl. Phys. Lett. **74,** 1373–1375 (1999).
24. A. V. Nesterov and V. G. Niziev, "Propagation features of beams with axially symmetric polarization," J. Opt. B: Quantum Semiclassical Opt. **3,** 215–219 (2001).
25. S. Solimeno, B. Crosignani, and P. DiPorto, *Guiding, Diffraction and Confinement of Optical Radiation* (Academic Press, 1986).